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Estimating Production Functions in Differentiated-Product Industries with Quantity Information and External Instruments*

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Abstract

This paper develops a new method for estimating production-function parameters that can be applied in differentiated-product industries with endogenous quality and variety choice. We take advantage of data on physical quantities of outputs and inputs from the Colombian manufacturing survey, focusing on producers of rubber and plastic products. Assuming constant elasticities of substitution of outputs and inputs within firms, we aggregate from the firm-product to the firm level and show how quality and variety choices may bias standard estimators. Using real exchange rates and variation in the "bite" of the national minimum wage, we construct external instruments for materials and labor choices. We implement a simple two-step instrumental-variables method, first estimating a difference equation to recover the materials and labor coefficients and then estimating a levels equation to recover the capital coefficient. Under the assumption that the instruments are uncorrelated with firms' quality and variety choices, this method yields consistent estimates, free of the quality and variety biases we have identified. Our point estimates differ from those of existing methods and changes in our preferred productivity estimator perform relatively well in predicting future export growth.

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1 Introduction

A central challenge in estimating production functions is to estimate the elasticities of real output with respect to real inputs, unconfounded by differences in prices across firms. Estimates of these elasticities are key to constructing standard measures of total factor productivity (TFP), the most commonly used metric of firm performance. They are also important for estimating markups in the influential method of Hall (1986) and De Loecker and Warzynski (2012). As recently emphasized by Bond et al. (2020), that method requires an elasticity of real output, not of sales or value-added, in order to generate informative estimates of markups.

Two difficulties in estimating such elasticities have received particular attention. First, prices and physical quantities of outputs and inputs are usually not observed at the firm (or plant) level. The most common solution is to regress sales (or value-added), deflated by a sector-level price index, on material expenditures and other inputs, similarly deflated. It has long been recognized that the resulting estimates may reflect idiosyncratic variation in market power at the firm level (Klette and Griliches, 1996; Foster et al., 2008, 2016; De Loecker and Goldberg, 2014). Second, firms may choose variable inputs after observing shocks to their own productivity in a given period, generating a positive correlation between unobserved productivity shocks and the input choices — the familiar "transmission bias" problem, first noted by Marschak and Andrews (1944).[1]

Information on prices and quantities at the firm-product level, while still uncommon, is increasingly available and has enabled researchers to make progress on the first issue. Focusing on eleven homogeneous products in the US Census of Manufactures, Foster et al. (2008) estimate regressions with physical output quantities on the left-hand side, yielding output elasticities arguably purged of demand-side influences. Although Foster et al. (2008) do not use physical quantities of inputs, in cases where inputs are homogeneous and quantities are observed it is straightforward to extend their approach and put physical quantities of inputs on the right-hand side (Atalay, 2014).

But as suggested by Katayama et al. (2009), Grieco and McDevitt (2016), Atkin et al. (2019), Jaumandreu and Yin (2018) and others, using physical quantities may be misleading in differentiated-product industries where the quality and variety of outputs and inputs vary across firms and over time. If consumers value product quality and variety, then they should be

[1] For reviews, see Griliches and Mairesse (1998), Bartelsman and Doms (2000), Ackerberg et al. (2007), Van Biesebroeck (2008), Syverson (2011), De Loecker and Goldberg (2014), Section 2 of Ackerberg et al. (2015), and Section 2.2.1 of Verhoogen (2020).

incorporated in our notion of real output; similarly, if input quality and variety matter for real output, then they should be incorporated in real inputs. But once one accepts these propositions, estimates using only physical units may be subject to what we call *quality* and *variety biases*. For instance, a firm may take advantage of an increase in capability to produce fewer physical units of higher-quality goods (for a given set of inputs), generating a negative bias in output elasticities estimated using only physical output quantities. If more-productive firms tend to use higher-quality inputs and this choice is correlated with the physical quantity of inputs used, then another form of bias arises, with the direction depending on the sign of the input quality-quantity correlation. Similar biases can also result from the endogenous choice of variety by firms, or from exogenous shocks to product appeal or input quality, if firms' choices of physical units of inputs respond.

In this paper, we develop a new approach to estimating output elasticities that takes advantage of quantity information, that is arguably not subject to quality or variety biases, and that also addresses the transmission-bias problem. The method can be applied in horizontally and vertically differentiated industries with multi-product firms and requires relatively weak theoretical assumptions on the nature of demand and market competition. We implement it in data from the Colombian manufacturing survey, which contains information on prices and quantities of both inputs and outputs, focusing (for reasons discussed below) on producers of rubber and plastic products.

The paper makes three main contributions. The first is to highlight conceptually how estimates of output elasticities based on physical quantities may be misleading in industries where quality and variety vary differentially by firm over time. As in almost all similar datasets, the mapping between specific inputs and specific outputs within the firm is unobserved.[2] Our approach is to aggregate from the firm-product to the firm level. It is not possible to do this aggregation in a theory-free way; any aggregation embeds assumptions, implicit or explicit, about consumer and firm behavior. Here we assume that outputs and inputs, respectively, have constant elasticities of substitution (CES) within firms. We place minimal constraints on substitution elasticities across firms. Following common practice, we assume that (firm-level aggregate) materials, labor, and capital combine in Cobb-Douglas fashion. Although restrictive, the within-firm CES structure is convenient in that it allows us to express the change in each aggregate as the sum of an observ-

[2]We are aware of only two exceptions with a substantial number of firms, a dataset on Bangladeshi garment firms used by Cajal Grossi et al. (2019) and one on Chinese steel firms used by Brandt et al. (2018).

able quantity index and unobservable terms capturing quality and variety. This in turn makes transparent how differences in quality and variety may bias standard estimates. Empirically, we will show that our estimates are robust to using other common aggregators at the firm level.

Our second contribution is to address transmission bias by introducing "external" instruments capturing exogenous variation in input prices at the firm level. The idea that external instruments in general, and input prices in particular, would be an attractive solution to the transmission-bias problem has been "in the air" for many years, at least since the landmark review by Griliches and Mairesse (1998).[3] Several recent papers have acknowledged that factor prices would be natural instruments, but have argued that it would be difficult to find truly exogenous variation at the firm level.[4] In the absence of credible external instruments, two approaches have dominated the recent literature. One has been to construct a proxy for unobserved productivity by inverting either an investment-demand or a materials-demand equation, which requires a monotonic relationship between the productivity term (assumed to be scalar) and investment or materials, conditional on other observables (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Wooldridge, 2009; De Loecker, 2011; Doraszelski and Jaumandreu, 2013, 2018; Ackerberg et al., 2015; Gandhi et al., 2020). Another approach has been to construct "internal" instruments using lagged values of inputs (Chamberlain, 1982; Anderson and Hsiao, 1982; Arellano and Bond, 1991; Arellano and Bover, 1995; Blundell and Bond, 1998, 2000).[5] In this panel-data approach, the most successful strategy has been the "System GMM" estimator of Arellano and Bover (1995) and Blundell and Bond (1998, 2000), which supplements an equation in first differences, using lagged levels as instruments, with an equation in levels, using lagged differences as instruments.

Our aggregation strategy requires firm-specific normalizations, which we absorb with a firm effect in our main estimating equation. As a consequence, the proxy-variable strategy is not an attractive option, because the firm effect would violate the required monotonicity assumption

[3] Griliches and Mairesse (1998) write that the "future" of production-function estimation lies in "find[ing] (instrumental) variables that have genuine information about factors which affect firms differentially as they choose their input levels" (p. 198) and describe using "factor prices ... as instrumental variables to identify the parameters of interest" as an "obvious" solution. Nerlove (1963) is an early paper using an external instrument — electricity prices — in production-function estimation.

[4] For instance, Ackerberg et al. (2015, p, 2418, fn 3) write: "if one observed exogenous, across-firm-variation in all input prices, estimating the production function using input price based IV methods might be preferred to OP/LP [Olley-Pakes/Levinsohn-Petrin] related methodology (due to fewer auxiliary assumptions)." But they also note that "the premise of most of this [proxy-variable] literature is that such variables are either not available or not believed to be exogenous." See also Ackerberg et al. (2007, p. 4208) and Gandhi et al. (2020, Sec. 6.1).

[5] The two dominant approaches are themselves related, since the proxy-variable methods often use lagged levels as instruments, as discussed in Ackerberg et al. (2015, Sec. 4.3.3).

(Ackerberg et al., 2015). We instead build on the panel-data approach, which more easily accommodates the firm effect. But rather than include further and further lags as instruments, which is commonly done but may raise concerns about instrument strength, we include a parsimonious set of lags and two external instruments capturing exogenous variation in input prices. To construct the materials-price instrument, we first use exchange rates to predict import-price changes at the product level, omitting one firm at a time in making the predictions. We then use the lagged product composition of a firm's imports, in conjunction with the "leave one out" predicted import-price changes, to construct a firm-specific predicted import price index. To construct the wage instrument, we interact changes in the national minimum wage, which saw large increases in real terms over our study period, with an indicator for how binding the change was likely to be on particular firms (the "bite" of the minimum wage), defined as the lagged ratio of the minimum wage to the average wage for permanent employees in the firm. We will see that the external instruments are helpful in alleviating (if not completely removing) weak-instrument concerns.

In estimating the coefficient on capital, we face a well-known difficulty: methods with transformations to remove firm effects — either first-differencing or deviating from firm means — tend to yield implausibly low estimates.[6] The most common explanation is that transformations to remove the firm effect exacerbate the attenuation bias due to measurement error; other explanations, discussed below, are also possible (Griliches and Mairesse, 1998). Others have found that this problem persists when instrumenting the change in capital with lagged levels (see e.g. Ornaghi (2006)). Fundamentally, the issue is that much of the genuine variation in capital stock is cross-sectional; the within-firm evolution of capital stock — and, in particular, of utilized capital — is very difficult to measure well. The main existing approaches, proxy-variable methods and System GMM, both rely to some extent on cross-sectional variation to estimate the capital coefficient.

Our third contribution is an approach to estimation that allows us both to absorb the firm effect when estimating the materials and labor coefficients and to take advantage of cross-sectional variation to estimate the capital coefficient. In the broad spirit of System GMM, we combine a difference equation, using lagged levels as instruments, with a levels equation, using lagged differences as instruments. But rather than estimating the equations simultaneously using GMM, we estimate them separately in what we call a two-step instrumental-variables (TSIV) procedure.

[6] See e.g. Griliches and Mairesse (1998), Ackerberg et al. (2007), and Ackerberg et al. (2015).

In the first step, we first-difference and use the external instruments described above, together with a parsimonious set of lagged levels, to recover the coefficients on materials and labor. In the second step, we use the first-step estimates of the materials and labor coefficients and impose an additional assumption that ensures orthogonality between the lagged difference in log capital and the firm effect; this allows us to use the lagged difference of capital as an instrument in an IV model in levels.[7] If the model is correctly specified, the TSIV estimator is less efficient than simultaneous GMM estimation of the difference and levels equations, but it has the advantage that the materials and labor coefficient estimates are robust to misspecification of the levels equation (Kripfganz and Schwarz, 2019).

The TSIV procedure yields plausible point estimates: we find materials and labor coefficients of approximately .4, and a capital coefficient of approximately .2. The fact that constant returns to scale approximately hold is reassuring. Although the standard errors are large enough that the differences with standard estimators are generally not statistically significant, the point estimates display some interesting patterns. The materials coefficient is lower than, and the labor coefficient larger than, the coefficients from (a) naive OLS estimation using revenues for output and materials expenditures for material input, (b) the Olley and Pakes (1996) and Levinsohn and Petrin (2003) proxy-variable methods, and (c) standard System GMM.[8] Somewhat surprisingly, our estimates (including for capital) are similar to those from OLS in levels using our constructed quantity aggregates.

The main goal of this paper is to provide a new approach to estimating output elasticities, which are useful for several purposes, including calculating markups. But it is natural to ask how TFP measures constructed from our estimates perform relative to other TFP measures commonly used in the literature. We face a choice between a revenue-based TFP and a quantity-based TFP measure based on our quantity indexes. We note that even with our improved estimates of output elasticities, neither measure corresponds directly to technical efficiency. Our revenue-based TFP may reflect pure output or input price differences, and our quantity-based TFP may be biased by quality and variety choices (even if the output-elasticity estimates are not). We favor using revenue-based TFP, and keeping in mind that it may capture price differences as well as technical efficiency. To compare with TFP estimates from other methods, we examine the extent to which

[7] The second-step approach is akin to that of Collard-Wexler and De Loecker (2016).

[8] Our materials estimate is quite similar to the one we obtain by applying Gandhi et al. (2020) in our data, and our labor coefficient is somewhat smaller.

estimated changes in revenue-based TFP predict future changes in export performance, and we find that our estimates compare well to standard methods.

In addition to the studies cited above, this paper is related to several branches of literature. It is perhaps most closely related to a small number of studies on production-function estimation in multi-product firms using information at the firm-product level. This literature has dealt in different ways with the lack of an observed mapping between inputs and outputs in multi-product firms. One strategy has been to focus on single-product firms, for which the mapping is clear, and (in some cases) to do a selection correction for the fact that they may not be representative (Foster et al., 2008; De Loecker et al., 2016; Garcia-Marin and Voigtländer, 2019; Balat et al., 2018; Blum et al., 2018).[9] Recent work by Dhyne et al. (2020a,b) develops an alternative strategy of estimating simultaneous equations relating output of each good to firm-level input usage and the output levels of other goods. This strategy requires as many proxies as products and a multi-dimensional generalization of the monotonicity condition, and it tends to reject the hypothesis that single-product production is a good approximation for multi-product production. Another approach has been to use estimates of demand elasticities and profit-maximization conditions to infer the allocation of inputs to outputs that would be implemented by optimizing firms (Orr, 2020; Valmari, 2016).[10] Our strategy, by contrast, is to aggregate both outputs and inputs to the firm level. Previous papers that have aggregated from the firm-product to the firm level include Eslava et al. (2004, 2013), Ornaghi (2006), Doraszelski and Jaumandreu (2013), Smeets and Warzynski (2013) and Bas and Paunov (2020).[11] These papers do not use CES aggregators, nor do they show how explicitly how quality or variety differences enter firm-level price or quantity indexes. Our approach builds on an extensive literature using CES functions in addressing other questions, including Feenstra (1994), Hsieh and Klenow (2009), Hottman et al. (2016), and Redding and Weinstein (2020).

This paper is also related to studies that explicitly consider differences in the quality of outputs or inputs in a production-function context. Melitz (2000), Katayama et al. (2009), and Grieco et

[9]Foster et al. (2008) include in their sample only firms in which one product makes up more than 50% of revenues, thus essentially focusing on single-product firms. De Loecker et al. (2016) implement a modified version of the Ackerberg et al. (2015) proxy-variable method and focus on calculating mark-ups at the firm-product level in Indian data. In Chilean data, Garcia-Marin and Voigtländer (2019) use firm-product-level markups calculated along the same lines to infer marginal costs and to relate them to firms' export behavior.

[10]See also Gong and Sickles (2019) and Forlani et al. (2016).

[11]Dhyne et al. (2020a) and Garcia-Marin and Voigtländer (2019) employ similar aggregations in parts of their analyses.

al. (2016) propose estimators that take quality differences into account in settings where product-level information is not observed; the lack of direct price and quantity data means that they must rely on more restrictive theoretical assumptions than we do here. Fox and Smeets (2011) show that including detailed indicators of labor quality significantly reduces the dispersion of estimated productivities across firms in Denmark, but they do not have product-level information on outputs or inputs. For the most part, the literature exploiting information at the firm-product level does not explicitly take into account quality or variety differences. Exceptions include De Loecker et al. (2016) and Eslava and Haltiwanger (2020), who use control-function approaches to address potential quality and variety biases.[12] Two recent papers take advantage of detailed product characteristics in particular sectors. Focusing on outpatient dialysis centers in the US, Grieco and McDevitt (2016) find that firms trade off quality and quantity of care, suggesting that measures of performance based solely on quantity can be misleading. In an Egyptian rug cluster, Atkin et al. (2019) collect direct measures of rug quality and producer performance under laboratory conditions and also find that purely quantity-based measures of performance are misleading. Such direct measures of product quality are clearly very valuable for estimating firm performance, but unfortunately they are rarely available. We view our method as being most useful in settings where product prices and quantities are available but detailed product characteristics are not.

Relative to the existing literature, our approach has costs and benefits. On one hand, some of our assumptions are stronger than those of other methods. We impose an unusually strong (but testable) restriction on the evolution of unobserved productivity, discussed below. We require exclusion restrictions for our instruments. We do not consider firms' endogenous investments in raising productivity, as do for instance Doraszelski and Jaumandreu (2013, 2018). The within-firm CES assumptions are restrictive (although the empirical patterns are robust to using other aggregators). On the other hand, we are able to avoid some strong assumptions required by other methods. We do not need a scalar monotonicity condition to ensure invertibility of an investment or materials-demand function as in proxy-variable methods. Although we presume some maxi-

[12]De Loecker et al. (2016) describe their approach as addressing "input price bias" and "output price bias" and do not explicitly address what we call quality and variety biases. They put flexible functions of output prices and market shares in a control function for input demand and put physical quantities of output on the left-hand side. Arguably, this approach completely removes quality biases only in the special case where input and output quality are perfectly correlated, which is unlikely to hold exactly in practice, and does not address what we call variety biases. Eslava and Haltiwanger (2020) also use CES aggregation, but they do so in the context of joint GMM estimation of production and demand functions, which requires CES across as well as within firms, while here we do not need to impose a particular demand structure across firms.

mizing behavior on the part of firms to justify the CES aggregation, we do not need first-order conditions for aggregate materials or labor to hold exactly as in Doraszelski and Jaumandreu (2013, 2018) and Gandhi et al. (2020). We can remain agnostic about cross-firm demand elasticities. Relative to the panel-data literature, we are able to reduce the reliance on lagged internal instruments. We can also relax the assumptions required for the levels equation in System GMM, and do not need them at all if we are only interested in the output elasticities with respect to materials and labor. We explicitly consider output and input quality and variety differences, as relatively few other papers have done. While there are trade-offs, we believe that, on balance, our method represents an attractive alternative to existing methods in differentiated-product industries where quantity information and external instruments are available.

The next section develops our econometric strategy. Section 3 describes the data we use and our motivation for focusing on producers of rubber and plastic products. Section 4 presents our baseline estimates of output elasticities, and Section 5 conducts several robustness checks. Section 6 compares our coefficient estimates to those of other common estimation methods. Section 7 constructs productivity measures using our coefficient estimates and examines how well they do, relative to existing measures, in predicting future export performance. Section 8 concludes.

2 Econometric Strategy

This section first presents the theoretical framework that underpins our firm-level aggregation and estimating equations. We begin on the demand side (Subsection 2.1) and then turn to the production side (Subsection 2.2) and rewrite the production function using decompositions of our output and input aggregates, which makes clear how endogenous quality and variety choices may bias standard estimates (Subsection 2.3). We then present our two-step IV (TSIV) strategy (Subsection 2.4). Full derivations are in Appendix A.

2.1 Demand

The first task is to construct a measure of real output at the firm level — firm-level sales deflated by an appropriate firm-specific price index. In differentiated-good industries, any price index necessarily embeds assumptions about how a firm's products enter consumers' utility. Here we follow Hottman et al. (2016) and others in imposing constant elasticity of substitution of products within firms. This is restrictive, but unlike much of the existing literature we do not need to make strong assumptions about the elasticity of substitution of products across firms. (We will also show (in Section 5.1 below) that the empirical patterns are robust to using other common aggregators.)

We assume that a representative consumer has the following utility function:

$$U_t = U(\widetilde{Y}_{1t}, \widetilde{Y}_{2t}, ..., \widetilde{Y}_{It}) \quad \text{where} \quad \widetilde{Y}_{it} = \left[\sum_{j \in \Omega^y_{it}} (\varphi_{ijt} Y_{ijt})^{\frac{\sigma^y_i - 1}{\sigma^y_i}} \right]^{\frac{\sigma^y_i}{\sigma^y_i - 1}} \tag{1}$$

Here i, j and t index firms, products (outputs), and periods (years), I is the total number of firms, Y_{ijt} is physical quantity of output, σ^y_i is the elasticity of substitution between outputs, specific to firm i, and Ω^y_{it} is the set of products sold by the firm. The φ_{ijt} terms are demand shifters that can be interpreted as product appeal or quality, which may reflect endogenous choices of the firm (e.g. physical attributes of goods) or external factors (e.g. exogenous fashion trends). We assume that $U(\cdot)$ is quasi-concave and weakly separable in its arguments. We follow common practice and assume that $\sigma^y_i > 1$. Although the consumer optimization problem would remain well-behaved as long as $\sigma^y_i > 0$,[13] the stronger $\sigma^y_i > 1$ ensures that the representative consumer will purchase more units of a good that increases in appeal, which seems realistic in our context.[14]

The assumption of weak separability and the homotheticity of $\widetilde{Y}_{it}$ imply that the consumer's optimization problem can be solved in two stages, first choosing the quantity of each variety from firm i, Y_{ijt}, to minimize the cost of acquiring each unit of $\widetilde{Y}_{it}$ and second choosing $\widetilde{Y}_{it}$ to maximize utility. Assuming the consumer optimizes in the first stage, the price required to purchase one unit of $\widetilde{Y}_{it}$ is:

$$\widetilde{P}_{it} = \left[\sum_{j \in \Omega^y_{it}} \left(\frac{P_{ijt}}{\varphi_{ijt}} \right)^{1-\sigma^y_i} \right]^{\frac{1}{1-\sigma^y_i}} \tag{2}$$

This is the price index that sets $\widetilde{P}_{it}\widetilde{Y}_{it} = R_{it}$, where R_{it} is the consumer's total expenditures on goods of firm i, which are also the firm's revenues. Note that it is quality-adjusted: conditional

[13] See Appendix A.1. For the knife-edge Cobb-Douglas case of $\sigma^y_i = 1$, we would need an additional condition to ensure that the optimization problem remains well-behaved.

[14] As noted by Redding and Weinstein (2020), $\sigma^y_i > 1$ is sufficient to ensure that products are "connected substitutes" in the sense of Berry et al. (2013) and hence that the demand system is invertible. This is a sufficient condition, not a necessary one, and our method could be implemented in settings with a greater degree of complementarity between products, but for reasons of realism and convenience we maintain the standard assumption.

on the price of a given output, higher output quality reduces the value of the index.

An attractive feature of our approach is that we do not need to impose further assumptions on demand. The assumption of quasi-concavity implies that there is a unique demanded bundle, given by:

$$\widetilde{Y}_{it} = D_{it}(\widetilde{P}_{1t}, ..., \widetilde{P}_{It}, C_t) \quad \text{for } i = 1, 2, ..., I \tag{3}$$

where C_t is total consumption in period t. The demand for the output aggregate of a given firm depends only on the firm's own aggregate price index, the price indexes of other firms, and total consumption. We can leave the $D(\cdot)$ function unspecified.

The within-firm CES assumption allows us to decompose changes in the firm-specific price index in a particularly convenient way. Let Ω^{y*}_{it} be firm i's common outputs between $t-1$ and t (i.e. $\Omega^{y}_{it-1} \cap \Omega^{y}_{it}$), R^*_{it} be the consumer's expenditures on common goods (i.e. common-goods revenue for the firm), $\widetilde{P}^*_{it}$ and $\widetilde{Y}^*_{it}$ be the price and quantity indexes for common goods analogous to $\widetilde{P}_{it}$ and $\widetilde{Y}_{it}$, S^{y}_{ijt} be the consumer's expenditure share on product j among all products produced by firm i, and S^{y*}_{ijt} be the corresponding share among common goods.[15] Following Sato (1976), Vartia (1976), Feenstra (1994) and Redding and Weinstein (2020), it is straightforward to show (see Appendix A.1) that the log change in the firm-specific price level can be expressed as:

$$\ln\left(\frac{\widetilde{P}_{it}}{\widetilde{P}_{it-1}}\right) = \underbrace{\sum_{j\in\Omega^{y*}_{it}} \delta_{ijt} \ln\left(\frac{P_{ijt}}{P_{ijt-1}}\right) - \sum_{j\in\Omega^{y*}_{it}} \delta_{ijt} \ln\left(\frac{\varphi_{ijt}}{\varphi_{ijt-1}}\right)}_{=\ln\left(\frac{\widetilde{P}^*_{it}}{\widetilde{P}^*_{it-1}}\right)} - \frac{1}{\sigma^y_i - 1} \ln\left(\frac{\chi^y_{it-1,t}}{\chi^y_{it,t-1}}\right) \tag{4}$$

where:

$$\delta_{ijt} = \frac{\left(\frac{S^{y*}_{ijt} - S^{y*}_{ijt-1}}{\ln S^{y*}_{ijt} - \ln S^{y*}_{ijt-1}}\right)}{\sum\limits_{j\in\Omega^{y*}_{it}} \left(\frac{S^{y*}_{ijt} - S^{y*}_{ijt-1}}{\ln S^{y*}_{ijt} - \ln S^{y*}_{ijt-1}}\right)}, \quad \chi^y_{it,t-1} = \sum_{j\in\Omega^{y*}_{it}} S^y_{ijt}, \quad \chi^y_{it-1,t} = \sum_{j\in\Omega^{y*}_{it}} S^y_{ijt-1} \tag{5}$$

The first term on the right-hand side of (4) is (the log of) the familiar Sato-Vartia index (Sato, 1976; Vartia, 1976); it is an observable weighted average of product-specific price changes for

[15] That is, $R^*_{it} = \sum_{\Omega^{y*}_{it}} P_{ijt} Y_{ijt}$, $\widetilde{P}^*_{it} = \left[\sum_{j\in\Omega^{y*}_{it}} (P_{ijt}/\varphi_{ijt})^{1-\sigma^y_i}\right]^{\frac{1}{1-\sigma^y_i}}$, $\widetilde{Y}^*_{it} = \left[\sum_{j\in\Omega^{y*}_{it}} (\varphi_{ijt} Y_{ijt})^{1-\sigma^y_i}\right]^{\frac{1}{1-\sigma^y_i}}$ $S^{y*}_{ijt} = \frac{P_{ijt} Y_{ijt}}{R^*_{it}}$ for $j \in \Omega^{y*}_{it}$, and $S^y_{ijt} = \frac{P_{ijt} Y_{ijt}}{R_{it}}$.

common goods, with the "Sato-Vartia weights" δ_{ijt}. The second term is a weighted average of changes in (unobservable) product quality, again using the Sato-Vartia weights. Intuitively, increases in product quality tend to reduce the price index, other things equal.[16] Together, the first and second terms are equal to the log change in the common-goods price index, $\widetilde{P}^*_{it}$. The third term is an adjustment for entry and exit of products, first introduced by Feenstra (1994). Increases in product variety also tend to reduce the price index.[17] Although the σ^y_i term is unobservable, the $\chi^y_{it-1,t}$ and $\chi^y_{it,t-1}$ terms (which capture the common-goods shares of total firm revenues in periods $t-1$ and t) are observable.

Appendix A.1 further shows that the log change in the quantity index, $\widetilde{Y}_{it}$, can also be expressed in a simple decomposition:

$$\ln\left(\frac{\widetilde{Y}_{it}}{\widetilde{Y}_{it-1}}\right) = \underbrace{\sum_{j\in\Omega^{y*}_{it}} \delta_{ijt}\ln\left(\frac{Y_{ijt}}{Y_{ijt-1}}\right) + \sum_{j\in\Omega^{y*}_{it}} \delta_{ijt}\ln\left(\frac{\varphi_{ijt}}{\varphi_{ijt-1}}\right)}_{=\ln\left(\frac{\widetilde{Y}^*_{it}}{\widetilde{Y}^*_{it-1}}\right)} + \frac{\sigma^y_i}{\sigma^y_i - 1}\ln\left(\frac{\chi^y_{it-1,t}}{\chi^y_{it,t-1}}\right) \tag{6}$$

where $\widetilde{Y}^*_{it}$ is the quantity index for common goods. The first term is again the log of a Sato-Vartia index, this time for quantities, the second term captures improvements in product quality, and the third term captures increases in product variety.

It is worth noting that this within-firm CES approach nests the common approach of using firm revenues deflated by a sector-level price index to measure real ouput, as $\sigma^y_i \to \infty$.[18] In that sense, our aggregation method is strictly more general than the most commonly used one.

[16] Redding and Weinstein (2020), in a very different exercise, deal with the quality terms by assuming that the geometric average of product quality across products is time-invariant; our approach, by contrast, is to assume that they are orthogonal to the instruments we construct, as will be made clear below.

[17] For example, if no goods are dropped from $t-1$ to t but new goods are introduced, then $\chi^y_{it-1,t} = 1 > \chi^y_{it,t-1}$, which, since $\sigma^y_i > 1$ by assumption, implies a reduction in the price index. This reflects the fact that the utility function (1) embeds a taste for variety in the goods from a given firm.

[18] Put another way, as $\sigma^y_i \to \infty$ our approach would provide theoretical justification for the standard approach of deflating firm revenues by a sector-level price index. From (1) and (2), $\lim_{\sigma^y_i\to\infty}\widetilde{Y}_{it} = \sum_{j\in\Omega^y_{it}}\varphi_{ijt}Y_{ijt}$ and $\lim_{\sigma^y_i\to\infty}\widetilde{P}_{it} = \min_{j\in\Omega^y_{it}}(P_{ijt}/\varphi_{ijt})$. All goods purchased by the consumer have the same quality-adjusted price, call it $\breve{P}_t = P_{ijt}/\varphi_{ijt}\ \forall\ j \in \Omega^y_{it}$; goods with higher quality-adjusted prices are not purchased. Then $R_{it} = \sum_{j\in\Omega^y_{it}} P_{ijt}Y_{ijt} = \sum_{j\in\Omega^y_{it}}(P_{ijt}/\varphi_{ijt})\varphi_{ijt}Y_{ijt} = \breve{P}_t\widetilde{Y}_{it}$. Hence as $\sigma^y_i \to \infty$, deflating R_{it} by $\breve{P}_t$ yields real output. Our approach is more general in that it is theoretically justified also for $\sigma^y_i < \infty$.

2.2 Production

On the production side, we assume that real output, as defined above, is a function of capital, labor, and a firm-level CES materials aggregate, combining in Cobb-Douglas fashion:

$$\widetilde{Y}_{it} = \widetilde{M}_{it}^{\beta_m} L_{it}^{\beta_\ell} K_{it}^{\beta_k} e^{\omega_{it}+\eta_i+\xi_t+\epsilon_{it}} \quad \text{where} \quad \widetilde{M}_{it} = \left[\sum_{h \in \Omega_{it}^m} (\alpha_{iht} M_{iht})^{\frac{\sigma_i^m - 1}{\sigma_i^m}} \right]^{\frac{\sigma_i^m}{\sigma_i^m - 1}} \tag{7}$$

Here h indexes material inputs, Ω_{it}^m is the set of inputs purchased by the firm, M_{iht} is the quantity of each material input purchased, L_{it} is labor, and K_{it} is capital. We refer to α_{iht} as input quality, recognizing that it may reflect physical attributes of the inputs or characteristics of the technology used to combine them in production. It captures any differences across inputs in how much one physical unit of the input contributes to the input aggregate. The assumption that the production function is Cobb-Douglas in capital, labor, and materials is standard in the literature. In principle, our approach could be extended to other functional forms (e.g. translog), although other forms would require additional instruments. As on the output side, we assume the the firm-specific elasticity of substitution between inputs is greater than unity, $\sigma_i^m > 1$, which ensures that a firm consumes more of an input that increases in quality. In addition to being standard, this assumption is consistent with recent evidence at the micro level that intermediate inputs are typically substitutes (Dhyne et al., 2020b; Peter and Ruane, 2020); also, as discussed below, we believe that the assumption that inputs are subsitutes is particularly plausible in the subsectors we focus on.[19]

In the error term, ω_{it} is a firm-specific "ex ante" productivity shock that firms observe before choosing inputs but that is unobservable to the econometrician; η_i is a time-invariant firm effect; ξ_t is a sector- or economy-level shock; and ϵ_{it} is an "ex post" shock that is revealed after firms have chosen inputs (and hence is not "transmitted" to input choices). (We may also think of it as reflecting measurement error.) As is standard, we allow material inputs and labor to be adjustable in the short run and hence potentially correlated with the ex ante shock, ω_{it}, but assume that capital can be adjusted only with a lag of one period. We assume that the ex ante and ex post productivity shocks are uncorrelated with past values of inputs, but we allow for feedback from current shocks to future input choices (and from the ex ante shock to current choices). In the

[19] As on the output side, our method remains applicable, although with somewhat less intuitive implications, as long as $\sigma_i^m > 0$. See Appendix A.2.

language of Wooldridge (2010), we assume that the input choices are sequentially but not strictly exogenous. In addition, we assume that both ω_{it} and ϵ_{it} are serially uncorrelated. This assumption on ω_{it} is stronger than usually imposed, but it can be tested with standard methods (Arellano and Bond, 1991), and we will not reject the null of no serial correlation. Formally, we assume:

$$\begin{aligned} &E(\omega_{it}|\eta_i, K_{it}, K_{it-1}\widetilde{M}_{it-1}, L_{it-1}, \omega_{it-1}, \epsilon_{it-1}..., K_{i1}, \widetilde{M}_{i1}, L_{i1}, \omega_{i1}, \epsilon_{i1}) = 0 \\ &E(\epsilon_{it}|\eta_i, K_{it}, \widetilde{M}_{it}, L_{it}, K_{it-1}, \widetilde{M}_{it-1}, L_{it-1}, \omega_{it-1}, \epsilon_{it-1}..., K_{i1}, \widetilde{M}_{i1}, L_{i1}, \omega_{i1}, \epsilon_{i1}) = 0 \end{aligned} \tag{8}$$

where the conditioning on past values of the shocks implies a lack of serial correlation in the shocks. Here we assume that the firm effect, η_i, arguably captures within-firm persistence that might show up as serial correlation in models without fixed effects.[20]

It is important to note that the quality and variety of both outputs and inputs, represented by the quality terms φ_{ijt} and α_{iht} and the variety sets Ω^y_{it} and Ω^m_{it}, may be chosen endogenously by firms. Researchers have proposed a number of frameworks to analyze such choices; see for instance Kugler and Verhoogen (2012) on quality, and Eckel and Neary (2010), Bernard et al. (2011) and Mayer et al. (2014) on variety. Here we do not adopt a particular model of how firms make these choices, nor do we assume that the firm behaves optimally in making them. We need only that the choices are uncorrelated with our internal and external instruments, discussed in Subsection 2.4 below.

The derivations of the price and quantity indexes on the input side are analogous to those on the output (i.e. demand) side. Given the production function (7) (which is also weakly separable, with homothetic aggregate $\widetilde{M}_{it}$), the firm can be thought of as first choosing values of M_{iht} to minimize the cost of acquiring a given level of the aggregate input, $\widetilde{M}_{it}$, and then choosing optimal values of $\widetilde{M}_{it}$, L_{it} and K_{it}, given the demand function, (3). Firms are assumed to be price-takers on input markets. As discussed below, we believe that this assumption is reasonable in the subsectors we focus on.[21] Optimization in the first stage implies that the cost of purchasing

[20] Proxy-variable methods typically assume that ω_{it} follows a Markov process, which allows for flexible patterns of serial correlation, but they do not allow for a firm effect, as discussed below.

[21] Estimation of production functions when producers have oligopsony power in input markets has recently been considered by Rubens (2020). Although market power in input markets could potentially be accommodated in our framework, we leave this extension to future work.

one unit of the materials aggregate, $\widetilde{M}_{it}$, is:

$$\widetilde{W}_{it} = \left[\sum_{h \in \Omega_{it}^{m}} \left(\frac{W_{iht}}{\alpha_{iht}} \right)^{1-\sigma_i^m} \right]^{\frac{1}{1-\sigma_i^m}} \tag{9}$$

This is the price index that sets $\widetilde{W}_{it}\widetilde{M}_{it} = E_{it}^{m}$, where E_{it}^{m} is the firm's total expenditures on material inputs.

As on the output side, the CES assumption allows us to decompose input-price changes in a convenient way. Let Ω_{it}^{m*} be firm i's common inputs between $t-1$ and t (i.e. $\Omega_{it-1}^{m} \cap \Omega_{it}^{m}$), $\widetilde{W}_{it}^{*}$ and $\widetilde{M}_{it}^{*}$ be the price and quantity indexes for common inputs, E_{it}^{m*} be the firm's expenditures on common inputs, S_{iht}^{m} be the firm's expenditure share on input h, among all inputs purchased by firm i, and S_{iht}^{m*} the corresponding share among common inputs.[22] Appendix A.2 shows that the log change in the firm-specific input price level can be expressed as:

$$\ln\left(\frac{\widetilde{W}_{it}}{\widetilde{W}_{it-1}}\right) = \underbrace{\sum_{h \in \Omega_{it}^{m*}} \psi_{iht} \ln\left(\frac{W_{iht}}{W_{iht-1}}\right) - \sum_{h \in \Omega_{it}^{m*}} \psi_{iht} \ln\left(\frac{\alpha_{iht}}{\alpha_{iht-1}}\right)}_{\ln\left(\frac{\widetilde{W}_{it}^{*}}{\widetilde{W}_{it-1}^{*}}\right)} - \frac{1}{\sigma_i^m - 1} \ln\left(\frac{\chi_{it-1,t}^{m}}{\chi_{it,t-1}^{m}}\right) \tag{10}$$

where:

$$\psi_{iht} = \frac{\left(\frac{S_{iht}^{m*} - S_{iht-1}^{m*}}{\ln S_{iht}^{m*} - \ln S_{iht-1}^{m*}}\right)}{\sum\limits_{h \in \Omega_{it}^{m*}} \left(\frac{S_{iht}^{m*} - S_{iht-1}^{m*}}{\ln S_{iht}^{m*} - \ln S_{iht-1}^{m*}}\right)}, \quad \chi_{it,t-1}^{m} = \sum_{h \in \Omega_{it}^{m*}} S_{iht}^{m}, \quad \chi_{it-1,t}^{m} = \sum_{h \in \Omega_{it}^{m*}} S_{iht-1}^{m} \tag{11}$$

As for output prices, the first term is the log Sato-Vartia observable price change index for common goods; the second term is a weighted average of changes in input quality; and the third term is an adjustment for entry and exit of inputs.

As for output quantities, the change in the CES materials quantity aggregate can again be written as the sum of an observable Sato-Vartia quantity change index and unobservable terms

[22] That is, $E_{it}^{m*} = \sum_{h \in \Omega_{it}^{m*}} W_{iht} M_{iht}$, $\widetilde{W}_{it}^{*} = \left[\sum_{h \in \Omega_{t,t-1}^{m*}} (W_{iht}/\alpha_{iht})^{1-\sigma_i^m}\right]^{\frac{1}{1-\sigma_i^m}}$, $\widetilde{M}_{it}^{*} = \left[\sum_{h \in \Omega_{t,t-1}^{m*}} (\alpha_{iht} M_{iht})^{1-\sigma_i^m}\right]^{\frac{1}{1-\sigma_i^m}}$, $S_{iht}^{m*} = \frac{W_{iht} M_{iht}}{E_{it}^{m*}}$ for $h \in \Omega_{it}^{m*}$, and $S_{iht}^{m} = \frac{W_{iht} M_{iht}}{E_{it}^{m}}$.

capturing increases in variety and quality:

$$\ln\left(\frac{\widetilde{M}_{it}}{\widetilde{M}_{it-1}}\right) = \underbrace{\sum_{h\in\Omega_{it}^{m*}} \psi_{iht} \ln\left(\frac{M_{iht}}{M_{iht-1}}\right) + \sum_{h\in\Omega_{it}^{m*}} \psi_{iht} \ln\left(\frac{\alpha_{iht}}{\alpha_{iht-1}}\right)}_{\ln\left(\frac{\widetilde{M}^*_{it}}{\widetilde{M}^*_{it-1}}\right)} + \frac{\sigma_i^m}{\sigma_i^m - 1} \ln\left(\frac{\chi^m_{it-1,t}}{\chi^m_{it,t-1}}\right) \quad (12)$$

where $\widetilde{M}^*_{it}$ is the quantity index for common inputs. This approach again nests the standard approach of using expenditures deflated by a sector-level input price index as $\sigma_i^m \to \infty$; see footnote 18.

2.3 Deriving Estimating Equation

To integrate the CES output and input quantity decompositions (6) and (12) into the production function, (7), it is convenient to restate the decompositions in levels. Let lower-case letters represent logs and $\triangle$ a change from $t-1$ to t. Summing the differences in (6) and (12) over time within firms, with firm-specific normalizations $\widetilde{y}_{i0}$ and $\widetilde{m}_{i0}$, we have:

$$\widetilde{y}_{it} = \underbrace{\widetilde{y}_{i0} + \sum_{\tau=1}^{t} \sum_{j\in\Omega_{i\tau}^{y*}} \delta_{ij\tau} \triangle y_{ij\tau}}_{=:\widetilde{y}_{it}^{SV}} + \underbrace{\sum_{\tau=1}^{t} \sum_{j\in\Omega_{i\tau}^{y*}} \delta_{ij\tau} \ln\left(\frac{\varphi_{ij\tau}}{\varphi_{ij\tau-1}}\right)}_{=:q_{it}^{y}} + \underbrace{\left(\frac{\sigma_i^y}{\sigma_i^y - 1}\right) \sum_{\tau=1}^{t} \ln\left(\frac{\chi^y_{i\tau-1,\tau}}{\chi^y_{i\tau,\tau-1}}\right)}_{=:v_{it}^{y}} \quad (13)$$

$$\widetilde{m}_{it} = \underbrace{\widetilde{m}_{i0} + \sum_{\tau=1}^{t} \sum_{h\in\Omega_{i\tau}^{m*}} \psi_{ih\tau} \triangle w_{ih\tau}}_{=:\widetilde{m}_{it}^{SV}} + \underbrace{\sum_{\tau=1}^{t} \sum_{h\in\Omega_{i\tau}^{m*}} \psi_{ih\tau} \ln\left(\frac{\alpha_{ih\tau}}{\alpha_{ih\tau-1}}\right)}_{=:q_{it}^{m}} + \underbrace{\left(\frac{\sigma_i^m}{\sigma_i^m - 1}\right) \sum_{\tau=1}^{t} \ln\left(\frac{\chi^m_{i\tau-1,\tau}}{\chi^m_{i\tau,\tau-1}}\right)}_{=:v_{it}^{m}}$$

where we define the new variables under the underbraces, $\widetilde{y}_{it}^{SV}$, q_{it}^y, v_{it}^y, $\widetilde{m}_{it}^{SV}$, q_{it}^m, and v_{it}^m, to be equal to the indicated summations. Note that in defining variables in this way, we set the quality and variety terms q_{it}^y, v_{it}^y, q_{it}^m, and v_{it}^m to zero in the initial year and include the firm-specific normalizations as part of the "Sato-Vartia" quantity terms, $\widetilde{y}_{it}^{SV}$ and $\widetilde{m}_{it}^{SV}$.

Plugging these expressions into the production function, (7), and rearranging, we have:

$$\widetilde{y}_{it}^{SV} = \beta_m \widetilde{m}_{it}^{SV} + \beta_\ell \ell_{it} + \beta_k k_{it} + \eta_i + \xi_t + u_{it} \quad (14)$$

where:

$$u_{it} = (\beta_m v_{it}^m - v_{it}^y) + (\beta_m q_{it}^m - q_{it}^y) + \omega_{it} + \epsilon_{it} \tag{15}$$

This equation relates the Sato-Vartia output quantity index to the Sato-Vartia input quantity index (both observable, modulo the firm-specific normalizations), log capital, log labor, a firm effect, a year effect, and an error term that reflects variety and quality of outputs and inputs as well as the "ex ante" and "ex post" productivity shocks.

Writing the production function in this way helps to clarify two issues. The first is that simply using physical quantities for output and input may be problematic in a setting where quality or variety vary differently by firm over time, on the output side or the input side. The input choices $\widetilde{m}_{it}^{SV}$, ℓ_{it}, and k_{it} may be correlated with the unobserved quality and variety terms, q_{it}^m, q_{it}^y, v_{it}^m, and v_{it}^y, generating what we call *output-* or *input-quality bias*, or *output-* or *input-variety bias*. To fix ideas, suppose that firms produce a single product using a single material input, in which case $\widetilde{y}_{it}^{SV}$ and $\widetilde{m}_{it}^{SV}$ simplify to the physical quantities of output and input and the variety terms drop out. If producing one unit of a higher-quality output requires more physical units of labor, with all else equal, then there will be a positive correlation between ℓ_{it} and the q_{it}^y (and hence a negative correlation between ℓ_{it} and the $-q_{it}^y$ in the error term), generating a negative output-quality bias in the OLS estimate of β_ℓ. Biases may also arise from purely exogenous shocks to product appeal or input quality, if such shocks affect firms' input choices — for instance, if a firm's product becomes fashionable for reasons unrelated to the firm's actions but it expands production to take advantage of the increased demand, or if a supplier improves the quality of a purchased input without changing the price and this induces the firm to increase output.

Among multi-product, multi-input firms, biases could arise from changes in variety. For instance, if import-tariff reductions increase the set of input varieties available and induce firms to increase the variety of inputs purchased, the variety of outputs produced, and total output, as suggested by Goldberg et al. (2010) and Bas and Paunov (2020), one would expect a positive correlation between $\widetilde{m}_{it}^{SV}$ and v_{it}^m and a negative correlation between $\widetilde{m}_{it}^{SV}$ and $-v_{it}^y$, generating offsetting biases with ambiguous net effects. It is important to note that these quality and variety biases are distinct from transmission bias, *and might be present even if one had a perfect proxy for the ex ante productivity term, ω_{it}.*[23]

[23] Note also that including a proxy for input quality alone, as in De Loecker et al. (2016), does not solve all of

The second issue that equation (14) clarifies is why the scalar monotonicity assumption required by standard proxy-variable approaches is incompatible with our approach to aggregation. The leading proxy-variable approaches require a one-to-one relationship between a firm's underlying productivity and either investment or materials demand, conditional on other observables (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; De Loecker, 2011; Doraszelski and Jaumandreu, 2013, 2018; Ackerberg et al., 2015; Gandhi et al., 2020) As noted by Ackerberg et al. (2015), in models with a firm effect (here η_i) in addition to the ex ante productivity term (here ω_{it}), this assumption is unlikely to hold, since the firm effect introduces a second dimension of heterogeneity between firms.[24] In our case, we assumed the presence of a firm effect at the outset, in the production function, (7). But even if we had not, we would have to deal with the firm-specific normalizations $\widetilde{y}_{i0}$ and $\widetilde{m}_{i0}$ in (13), which we have folded into the levels of the observable Sato-Vartia quantity aggregates, $\widetilde{y}_{it}^{SV}$ and $\widetilde{m}_{it}^{SV}$. We impose a particular normalization in the second step of our estimation procedure below, but we feel that a strength of our approach is that we do not need to impose such an assumption in the first step when estimating the coefficients on materials and labor. We will pursue an approach more in the spirit of the panel-data literature, in part because it can more easily accommodate the fixed effect.

2.4 Two-Step IV Estimation Procedure

To estimate the production function, (14), we proceed in two steps, each implementing an IV model. In the first step, we first-difference to remove the firm effect and use lagged levels and external drivers of input price changes as instruments. We recover estimates of β_m and β_ℓ from this step, but we treat β_k as a nuisance parameter, in part because we do not believe there is sufficient signal in the within-firm changes in capital to estimate β_k credibly. In the second step, we incorporate the first-step estimates of β_m and β_ℓ in the levels equation and use the lagged difference of capital as an instrument for the level, in the spirit of the System GMM approach (Arellano and Bover, 1995; Blundell and Bond, 1998, 2000). To be clear about terminology: we refer the first *step* as the "differences" step and the second as the "levels" step; in each step, there is an IV model that has two *stages*.

the potential problems; there may still be correlation of physical input choices with *output* quality or with output or input variety.

[24] This issue is separate from the concern that firms may face heterogeneous constraints in input markets, which might break the monotonic relationship between productivity and materials or investment demand (Shenoy, forthcoming).

2.4.1 Differences (Step 1)

A common approach to estimating an equation with a firm effect would be to implement a "within" estimator in which all variables are deviated from firm-specific means. But the within estimator would require the time-varying firm-specific productivity terms, ω_{it} and ϵ_{it}, to be uncorrelated with all past and future values of the covariates — strict exogeneity in Wooldridge (2010)'s terminology — which would be violated if productivity shocks affect future input choices. Instead, we first-difference to remove the firm effect, yielding an estimator that remains consistent under sequential exogeneity (which we have assumed in (8)). From (14),

$$\triangle \widetilde{y}_{it}^{SV} = \beta_m \triangle \widetilde{m}_{it}^{SV} + \beta_\ell \triangle \ell_{it} + \beta_k \triangle k_{it} + \triangle \xi_t + \triangle u_{it} \tag{16}$$

where:

$$\triangle u_{it} = (\beta_m \triangle v_{it}^m - \triangle v_{it}^y) + (\beta_m \triangle q_{it}^m - \triangle q_{it}^y) + \triangle \omega_{it} + \triangle \epsilon_{it}$$

Note that in addition to removing the firm effect, η_i, the first-differencing eliminates the firm-specific normalizations in (13), $\widetilde{y}_{i0}$ and $\widetilde{m}_{i0}$, from the Sato-Vartia terms, $\widetilde{y}_{it}^{SV}$ and $\widetilde{m}_{it}^{SV}$. We refer to (16) as our difference equation.

Ordinary least squares (OLS) estimation of (16) would be subject to the quality and variety biases discussed above as well as the familiar transmission bias (i.e. firms observe ω_{it} before choosing the flexible inputs, inducing a positive correlation between $\triangle\omega_{it}$ and $\triangle\widetilde{m}_{it}^{SV}$ and $\triangle\ell_{it}$). Additionally, in first differences even pre-determined variables will in general be correlated with the error term: k_{it}, which appears in $\triangle k_{it}$, may be correlated with ω_{it-1} or ϵ_{it-1}, which appear in $\triangle u_{it}$. To address these concerns, we seek instruments that are correlated with $\triangle\widetilde{m}_{it}^{SV}$, $\triangle\ell_{it}$ and $\triangle k_{it}$ and uncorrelated with the error term, $\triangle u_{it}$. If u_{it} is serially uncorrelated in levels, as we have assumed, then u_{it-2} and further lags are uncorrelated with $\triangle u_{it}$ and input levels in $t-2$ and further back are valid instruments for $\triangle\widetilde{m}_{it}^{SV}$, $\triangle\ell_{it}$ and $\triangle k_{it}$. Below we will implement standard tests for serial correlation from Arellano and Bond (1991) and find that we do not reject the null hypothesis of no serial correlation.[25]

[25] Note that our approach differs from standard System GMM in the assumption on the productivity shocks. Blundell and Bond (1998, 2000) assume that the ex ante productivity shocks follow an AR(1) process and quasi-difference to purge the serial correlation prior to estimation. Under the (testable) assumption of no serial correlation, this quasi-differencing is not necessary.

A widely recognized concern with using lagged levels as instruments, however, is that they may be only weakly correlated with current differences (Griliches and Mairesse, 1998; Blundell and Bond, 1998, 2000; Bun and Windmeijer, 2010; Hayakawa and Qi, 2020).[26] Including further and further lags may exacerbate the weak-instrument problem. Testing for weak instruments is complicated here by the presence of multiple endogenous covariates and the potential for heteroskedastic errors (which we would not feel justified in assuming away). This is a frontier area of econometric theory and there is no consensus in the literature on the right diagnostic tests to use in such cases.[27] Two tests are commonly reported in practice. Sanderson and Windmeijer (2016) propose an improved version of a test first suggested by Angrist and Pischke (2009), which is appropriate for estimation and inference on each of multiple endogenous regressors, as we are interested in here.[28] Also commonly reported is the Kleibergen and Paap (2006) Wald statistic, an analogue of the Cragg and Donald (1993) statistic applicable in non-homoskedastic settings.[29] We will report both the Sanderson-Windmeijer and Kleibergen-Paap Wald statistics below. The statistics will give reason to be concerned about the weakness of the internal instruments.

Our strategy for strengthening the instrument set is to incorporate external instruments capturing exogenous variation in the prices of materials and labor. To construct the materials-price instrument, we proceed in two steps, taking advantage of detailed Colombian trade-transactions data merged with the Colombian manufacturing survey. We first use real-exchange-rate movements to predict import-price movements at the product-year level.[30] We then use information on the product composition of each firm's imports to aggregate the predictions to the firm-year level. To ensure that the predicted import index is uncorrelated with u_{it-1}, which appears in $\triangle u_{it}$ in (16), we run "leave one out" regressions (one for each firm, omitting that firm's imports) to predict import-price changes and we use import product composition from $t-2$ to do the

[26] In the setting of cross-country growth regressions, Bazzi and Clemens (2013) and Kraay (2015) show that the instruments used in difference and system GMM estimators are weak and can suggest misleading inferences. See also the review by Bun and Sarafidis (2015).

[27] In a recent state-of-the-art review, Andrews et al. (2019) recommend the test of Montiel Olea and Pflueger (2013) in cases with a single endogenous regressor, but have no recommendation in cases with multiple endogenous regressors; see their footnote 4.

[28] The Sanderson-Windmeijer statistic adjusts for the fact that the endogenous covariates may themselves be highly correlated. The theoretical justification for it relies on an assumption of homoskedastic errors, but it is commonly reported even in non-homoskedastic settings.

[29] Although the Kleibergen and Paap (2006) Wald statistic is sometimes compared to the Stock and Yogo (2005) critical values (see e.g. Baum et al. (2007)), the validity of this comparison is unclear in the non-homoskedastic case, since the critical values are calculated under the assumption of homoskedasticity.

[30] Exchange-rate movements have previously been used as a source of identification in production-function estimation by Park et al. (2010), who use them as a source of variation in demand from export markets.

aggregation.

To be precise, we begin by defining real exchange rates (RERs) as:

$$RER_{ot} = NER_{ot}\left(\frac{CPI_{ot}}{CPI_{Col,t}}\right) \tag{17}$$

where o indexes import origins, NER_{ot} is the nominal exchange rate (Colombian pesos/foreign currency), CPI_{ot} is the consumer price index (CPI) in the origin, and $CPI_{Col,t}$ is the CPI of Colombia. Defined in this way, a real appreciation in country o is reflected in an increase in RER_{ot}. We consider the top 100 origins by Colombian import volume and label this set $\mathcal{O}$. We use n to index products defined at the 8-digit trade classification level, which do not map cleanly to products in the Colombian industrial classification, indexed by j and h above. We exclude machinery and equipment, which could arguably be considered capital rather than material imports; we also exclude petroleum and other mineral fuels.[31] For a particular imported input n, we calculate an average log RER change separately for each firm in our data, weighting by imports but leaving out the firm's own imports:

$$\triangle\overline{rer}_{nt,-i} = \sum_{o\in\mathcal{O}} \zeta_{ont-1,-i} \triangle \ln(RER_{ot}), \quad \text{where } \zeta_{ont-1,-i} = \frac{\mathcal{I}_{ont-1,-i}}{\sum_{o\in\mathcal{O}} \mathcal{I}_{ont-1,-i}} \tag{18}$$

Here $\mathcal{I}_{ont-1,-i}$ is the "leave-one-out" value of imports of product n from origin o in period $t-1$ for all firms except i. We then use these product-level average real-exchange-rate changes to predict import price changes at the product-year level, using the regression:

$$\triangle w^{imp}_{nt,-i} = \gamma_{st} \triangle \overline{rer}_{nt,-i} + \rho_{st} + \eta_{nt} \tag{19}$$

where $\triangle w^{imp}_{nt,-i}$ is the change in import n's log import price (averaged across origins using import weights) for imports of all firms except i,[32] and ρ_{st} is a sector-year effect. In our preferred specification, s indexes two digit trade sectors and we allow the coefficient on the exchange-rate term to vary by two-digit trade sector and year.[33] We run this leave-one-out regression

[31] That is, we exclude Harmonized System 2-digit categories 27, 84 and 85.

[32] That is, $\triangle w^{imp}_{nt,-i} = \sum_{o\in\mathcal{O}} \zeta_{ont-1,-i} \triangle w^{imp}_{ont,-i}$, where $\zeta_{ont-1,-i}$ is defined as in (18).

[33] In principle, we could include lags of the average real-exchange-rate changes in (19). But consistent with the literature on exchange-rate pass-through (see e.g. Campa and Goldberg (2005)), we have found that the effect of RER changes on import prices decays relatively quickly, within one year, and including further lags has little effect on the strength of our instrument, so we do not include them here.

separately for each firm i (using data from all firms in the customs data that can be linked to the manufacturing survey, not just those in the rubber and plastics sectors) and recover the predicted values, $\triangle \widehat{w}^{imp}_{nt,-i}$.

We then use firm i's product-level import shares as weights in constructing the average predicted import price change at the firm level:

$$\triangle \widehat{\overline{w}}^{imp}_{it} = \sum_{n \in \mathcal{N}} \theta_{int-2} \triangle \widehat{w}^{imp}_{nt,-i}, \quad \text{where } \theta_{int-2} = \frac{I_{int-2}}{\sum_{n \in \mathcal{N}} I_{int-2}} \tag{20}$$

Here I_{int-2} is imports by firm i of product n in period $t-2$ and $\mathcal{N}$ is the set of all imported products. For firms that did not import in $t-2$, we set $\triangle \widehat{\overline{w}}^{imp}_{it} = 0$. This average predicted import price change at the firm level, $\triangle \widehat{\overline{w}}^{imp}_{it}$, is our external instrument for $\triangle \widetilde{m}^{SV}_{it}$ in (16).[34]

To construct an external instrument for labor, we exploit the fact that the minimum wage in Colombia is high relative to the wage distribution and that it rose sharply over our sample period, especially in 1994-1999 and 2003-2009. (See Subsection 3.3 for institutional background.) We first construct a measure of the "bite" of the minimum wage — how binding it is expected to be on a particular firm — defined as:

$$B_{it} = \frac{MW_t}{W^{\ell}_{it}} \tag{21}$$

where MW_t is the national minimum wage (defined for monthly earnings and annualized multiplying by 12) and W^{ℓ}_{it} is firm-level average annual earnings per worker for permanent workers, calculated as the firm-level annual wage bill divided by average employment. Defined in this way, $B_{it} < 1$ and the closer the firm average wage is to the national minimum wage, the larger is B_{it}. We interact this measure of bite with the change in the national minimum wage, using bite from $t-2$ (again to avoid correlation with u_{it-1}, which appears in $\triangle u_{it}$ in (16)):

$$\triangle z_{it} = B_{it-2} * \triangle \ln(MW_t) \tag{22}$$

This predicted wage change, $\triangle z_{it}$, serves as an instrument for $\triangle \ell_{it}$ in (16). Other studies that have followed this strategy of interacting minimum wage changes with differences in their bite include Card (1992), Stewart (2002), and Cengiz et al. (2019).

[34] Exchange-rate movements may also affect export prices. We address this concern by constructing an analogous predicted export price index and including it as an additional covariate; see Section 5.2 below.

It is well known that the estimation of the coefficient on capital is problematic in first-differenced models, and indeed in any model that includes a firm effect. For example, in a first-differenced model using lagged levels as instruments, Ornaghi (2006) finds a negative coefficient on capital. Using a within estimator, Söderbom and Teal (2004) also find a negative relationship. It is common to attribute low estimates of the capital coefficient to measurement error in capital, the effect of which is exacerbated by transformations to remove the firm effect (Griliches and Mairesse, 1998; Ackerberg et al., 2015). In the Colombian manufacturing census, we do not observe capital utilization, and it seems likely that the capital measure we are able to construct, while standard, is a very noisy measure of capital in use. It may also be that in the presence of adjustment costs for capital, with firms investing in a lumpy way and the returns to capital accruing over long periods, changes in capital are not likely to show up immediately in changes in output (Griliches and Mairesse, 1998). Griliches and Mairesse (1998) recommend looking at longer differences, to reduce the role of noisy year-to-year fluctuations. But as noted above (footnote 33), the real-exchange-rate fluctuations that are the main source of exogenous variation in our predicted-import-price instrument have an effect on prices only in the relatively short term, typically 1-2 quarters, and the instrument has little explanatory power over longer periods. If we had an external instrument that generated large changes in capital on a year-by-year basis, it would help to improve the estimation of the capital coefficient, but we have not found such an instrument. In light of these issues, we conclude that we do not have sufficient signal in year-on-year capital changes to estimate β_k well in first-differences. In the first step, we focus on estimating β_m and β_ℓ and treat β_k as a nuisance parameter. In the next subsection, we present a different strategy for estimating β_k, in levels, incorporating "between" variation.

The key identifying assumptions for this first step, in differences, are that the instruments — the external instruments, $\triangle\widehat{\widetilde{w}}_{it}^{imp}$ and $\triangle z_{it}$, and the internal instruments, $\widetilde{m}_{it-2}^{SV}$, ℓ_{it-2}, and k_{it-2} — are uncorrelated with the error term in (16), $\triangle u_{it}$. Under these assumptions, we can recover consistent estimates of output elasticities with respect to materials and labor, $\widehat{\beta}_m$ and $\widehat{\beta}_\ell$. If one is only interested in $\widehat{\beta}_m$ and/or $\widehat{\beta}_\ell$, for instance to use in constructing markups in the method of Hall (1986) and De Loecker and Warzynski (2012), then one can stop at this step.

2.4.2 Levels (Step 2)

In the second step, we proceed in the broad spirit of System GMM (Arellano and Bover, 1995; Blundell and Bond, 1998, 2000) in using lagged differences as instruments in a levels equation.[35] The System GMM assumption that lagged differences are uncorrelated with the firm effect is equivalent to assuming that the correlation of the level of the inputs and the firm effect is constant over time, which Bun and Sarafidis (2015) refer to as the "constant correlated effects" assumption. Here, because of the availability of the external instruments in Step 1, we need this assumption only for capital, not for materials or labor. That is, we assume:

$$E(k_{it}\eta_i) = c_i \tag{23}$$

for some (potentially firm-specific) constant c_i. This assumption rules out correlation between a firm's time-invariant productivity and the evolution of its capital stock over time and is clearly restrictive. But we believe that it is plausible in our setting, where much of the within-firm variation appears to be due to measurement error.[36]

To proceed, we also need to take a stand on the firm-specific normalizations, $\widetilde{m}_{i0}$ and $\widetilde{y}_{i0}$, in (13). This amounts to choosing a base year for the firm-specific output and input price indexes, $\widetilde{P}_{it}$ and $\widetilde{W}_{it}$. Here we assume that these indexes are equal to unity in the first year that a firm appears in our data. In logs, since $r_{it} = \widetilde{y}_{it} + \widetilde{p}_{it}$ in every period, setting $\widetilde{p}_{i0} = 0$ implies $\widetilde{y}_{i0} = r_{i0}$; that is, the log output quantity index in a firm's first year in the panel is set equal to log revenues. Similarly, on the input side, setting $\widetilde{w}_{i0} = 0$ implies $\widetilde{m}_{i0} = e_{i0}$; the log input quantity index is set equal to log expenditures. These choices for the firm-specific normalizations preserve the cross-sectional variation in sales and expenditures present in the initial year for each firm.

Using $\widehat{\beta}_m$ and $\widehat{\beta}_\ell$ from Step 1 and recalling that $\widetilde{y}_{i0}$ and $\widetilde{m}_{i0}$ are included in the definitions of $\widetilde{y}_{it}^{SV}$ and $\widetilde{m}_{it}^{SV}$, equation (14) can be rewritten as:

$$\widetilde{y}_{it}^{SV} - \widehat{\beta}_m \widetilde{m}_{it}^{SV} - \widehat{\beta}_\ell \ell_{it} = \beta_k k_{it} + \xi_t + \breve{u}_{it} \tag{24}$$

[35] Despite this similarity, our approach differs from Blundell and Bond (2000) in that we use a different set of instruments, we do not allow for for serial correlation in ex ante productivity (and hence do not quasi-difference to eliminate it), and we do not estimate the difference and levels equations simultaneously by GMM. See also Section 6.

[36] Assumption (23) would also be justified if current investment were a function of current and past innovations in productivity, i.e. $\omega_{it}, \omega_{it-1}, ..., \omega_{i0}$, but not the fixed effect, η_i.

where the error term now includes the firm effect, η_i, and terms arising from estimation error in the first-step estimates:

$$\breve{u}_{it} = \eta_i + (\beta_m - \widehat{\beta}_m)\widetilde{m}_{it}^{SV} + (\beta_\ell - \widehat{\beta}_\ell)\ell_{it} + (\beta_m v_{it}^m - v_{it}^y) + (\beta_m q_{it}^m - q_{it}^y) + \omega_{it} + \epsilon_{it}$$

We assume that the lagged difference in capital, $\triangle k_{it-1}$, is uncorrelated with the quality and variety terms, q_{it}^m, q_{it}^y, v_{it}^m and v_{it}^y. Under this assumption and the constant correlated effects assumption, (23), $\triangle k_{it-1}$ is a valid instrument for k_{it} in (24).[37] Although the first-step estimation errors in $\widehat{\beta}_m$ and $\widehat{\beta}_\ell$ show up in $\breve{u}_{it}$, the consistency of the first-step estimates implies that they will not render the second-step estimates inconsistent.[38] They may, however, need to be taken into account in estimating the standard error on $\hat{\beta}_k$. As explained in more detail in Appendix A.3, if there is no correlation between $\triangle k_{it-1}$ and $\widetilde{m}_{it}^{SV}$ or ℓ_{it}, then no correction of the standard errors is required, but if there is correlation, then a correction needs to be applied. Below we report both the uncorrected and the corrected standard errors.

If the model is specified correctly, then estimating it in two steps potentially involves a loss of efficiency relative to simultaneous GMM estimation. But as recently pointed out by Kripfganz and Schwarz (2019) in a related context, an advantage of the two-step approach is that our first-step estimates of β_m and β_ℓ are robust to mis-specification in the second stage, and in particular to violations of the constant-correlated-effects assumption, (23).

3 Data, Institutional Background, and Descriptive Statistics

This section describes the main datasets we use, reviews the institutional background on the minimum wage in Colombia, explains the motivation for our choice of subsectors, and presents descriptive statistics for our sample. Additional details are in Appendix B.

[37] We also present results using $\triangle k_{it-2}$ as the instrument for k_{it}.

[38] This follows from the facts that $\text{plim}_{I\to\infty} \frac{1}{I}\sum_i \triangle k_{it-1}\left[(\beta_m - \widehat{\beta}_m)\widetilde{m}_{it}^{SV} + (\beta_\ell - \widehat{\beta}_\ell)\ell_{it}\right] = \text{plim}_{I\to\infty} \frac{1}{I}(\triangle k_{it-1}\widetilde{m}_{it}^{SV}) \cdot \text{plim}_{I\to\infty}(\beta_m - \widehat{\beta}_m) + \text{plim}_{I\to\infty} \frac{1}{I}(\triangle k_{it-1}\ell_{it}) \cdot \text{plim}_{I\to\infty}(\beta_\ell - \widehat{\beta}_\ell)$ (where I is the total number of firms) and, if the first-step estimates are consistent, then $\text{plim}_{I\to\infty}(\beta_m - \widehat{\beta}_m) = \text{plim}_{I\to\infty}(\beta_\ell - \widehat{\beta}_\ell) = 0$. See Section 4 of Kripfganz and Schwarz (2019) and in particular their footnote 19.

3.1 Annual Manufacturing Survey

We use information on sales, employment, wages, capital stock, inputs and outputs from the *Encuesta Anual Manufacturera (EAM,* Annual Manufacturing Survey), collected by the Colombian statistical agency, DANE. Data are reported at the plant level, and we aggregate them to the firm level — the level at which we observe imports and exports from trade transactions records (see below). In the sectors we focus on, nearly all firms have just one plant. We focus on data from the period 1994-2009.[39] Given that we will need at least two lags in our baseline specifications, our main period of analysis is 1996-2009.

The survey contains information on the values and physical quantities of all outputs produced and inputs consumed by each plant at the level of eight-digit product categories.[40] Because the survey is used to construct producer price indexes, DANE pays careful attention to the physical units of measurement for each product, and a given product is always reported using the same units. We calculate product prices at the firm level as unit values: $P_{ijt} = R_{ijt}/Y_{ijt}$, where R_{ijt} is the value of product k produced by firm i in year t and Y_{ijt} the corresponding quantity. Input prices are calculated analogously. Further details, including on the construction of capital stock, which uses a standard perpetual-inventory method, are in Appendix B.1. The fact that the survey contains, in principle, information on all material inputs is important because it responds to a criticism of IV methods, for instance by Ackerberg et al. (2015), that the exclusion restrictions for input-price instruments are likely to be violated if one observes only a subset of inputs.

3.2 Customs Records and Exchange Rates

The customs data contain information from the administrative records filled out by every Colombian importer or exporter for each international transaction, collected by the Colombian customs agency, DIAN. Information is available at the level of the firm, product code (8 digit), year, and country of origin (for imports) or destination (for exports). Imports and exports by the firm are merged with the EAM manufacturing data using firm identifiers according to the procedures established by DANE. Further details are in Appendix B.2.

[39]Prior to 1994, the EAM used different plant identifiers and it is often difficult to track plants over time. Although we use data from 1992-1993 when available in constructing firm-level capital stock, we do not focus on these years in the main analysis. Using procedures established by DANE to protect the confidentiality of the data, it is possible to link the customs data (see below) to the EAM only until 2009.

[40]The survey also reports information on outputs sold and inputs purchased, but we use the information on production and consumption to avoid timing issues that arise because firms hold inventories.

To calculate real exchange rates (RERs) by trading partner, we use nominal exchange rates and consumer price indexes (CPIs) from the International Financial Statistics (IFS) of the International Monetary Fund.[41] Appendix Figure A1 depicts the movements in real exchange rates (defined such that an increase reflects a real appreciation in the trading partner) for the 12 countries from which rubber and plastics producers purchased the most imports during the period of our analysis. We see that several of the most important import origins had significant RER fluctuations. Venezuela and Mexico, both major oil producers, had large real appreciations in 1995-2000 and large real depreciations subsequently. Indonesia suffered a major crisis accompanied by sharp real devaluation in 1997 (as did Argentina (not pictured) in 2001). Even the US and Eurozone countries, which were less volatile overall, experienced non-trivial variation in the RER relative to Colombia.

3.3 Minimum Wage

Despite wide variation in local labor market conditions, Colombia has a single national minimum wage. Over our study period, it was one of the highest in Latin America as a share of the median wage, and it increased significantly in real terms (Maloney and Nuñez Mendez, 2004; Mondragón-Vélez et al., 2010). As required by the Colombian constitution, increases for the coming year are negotiated in December by a tripartite commission including representatives from government, employer associations, and labor organizations. Prior to 1999, the target was commonly understood to be predicted inflation plus predicted productivity growth (Maloney and Nuñez Mendez, 2004; Hofstetter, 2006). In 1999, because of a recession, predicted inflation greatly exceeded actual inflation and the real value of the minimum wage increased by 7%. In addition, the Constitutional Court in Colombia ruled in 1999 that the minimum wage increase could not be lower than the previous year's inflation. As a result, the real value of the minimum wage continued to rise after 2000, remaining above 90% of the median wage through the end of our study period (Mondragón-Vélez et al., 2010). Appendix Figure A2 shows the steady increase of the real minimum wage over our study period. To illustrate the bite of the minimum wage, Appendix Figure A3 plots a histogram of real wages in 1998 for individuals who report working in firms with 10 or more employees in manufacturing in a Colombian household survey, the *Encuesta Nacional de Hogares*. The solid and dashed vertical lines represent the 1998 and 1999 minimum

[41] For a few countries with no information in the IFS, we gathered data directly from their central banks.

wages, respectively. We see that there was extensive bunching of wages at the minimum in 1998, and that a large share of manufacturing workers was directly affected by the 1999 minimum wage increase. The minimum wage is often used to index the wages of employees who earn above the minimum; as a consequence, increases in the real minimum wage are likely to have an effect on wages throughout the distribution (Mondragón-Vélez et al., 2010). Researchers have previously found disemployment effects of the minimum wage in Colombia, in contrast to several other countries in the region (Bell, 1997; Maloney and Nuñez Mendez, 2004); below we will also find negative employment effects.

3.4 Choice of Subsectors and Descriptive Statistics

Our method is most applicable in industries that meet several criteria. First, the ability to accommodate endogenous quality and variety choices is most valuable in sectors producing differentiated products, particularly those with substantial quality variation. Second, given that we assume that firms are price-takers in input markets, our method is most applicable in industries in which inputs, although they may differ in quality, are relatively non-specialized and substitutable within quality categories. Third, for our instrument for materials to be relevant, a substantial share of inputs in the industry must be imported, such that real-exchange-rate fluctuations have a significant effect on the input prices faced by firms.

In choosing subsectors that fit these criteria, we face a familiar trade-off. On one hand, we would like sample sizes to be as large as possible in order to increase the precision of our estimates. This clearly matters in our setting where the weakness of instruments is a concern. On the other hand, the wider the net that we cast, the more heterogeneous the included firms are likely to be. This trade-off is well recognized in the literature; see for instance the discussion in Dhyne et al. (2020a, Sec. 4.3). The issue is particularly salient because, as is standard in the literature, we will treat all firms in a single broad sector as having the same production-function coefficients.

Our approach in this paper is to focus on firms producing rubber and plastic products. These subsectors are adjacent in the ISIC revision 2 classification (with 3-digit codes 355 and 356, respectively) and are often classified together in a 2-digit sector, as for instance in Sector 36 ("Rubber and Plastic Products") of the U.N. Central Product Classification (CPC). Table 1 reports their main 8-digit outputs. For rubber, the main product is tires of different sorts. These can be understood to be differentiated products: they are sold under brand names — Goodyear and Michelin tires are produced in Colombia, for instance — and often for fairly specialized uses. For plastics, there is less concentration on a single type of product; output is distributed across various types of tubing, bags, sheets, films, and containers. But again, the products of the sector are typically differentiated and often targeted for specialized uses.

By contrast, the inputs of both subsectors can be viewed as commodities, or at least commodity-like — highly substitutable across suppliers even if they have quality differences. Table 2 reports the main 8-digit inputs. For rubber, the most important input is natural latex, from the bark of rubber trees. The second-most important input category is "rare metals in primary forms" (CPC product code 3423112) which includes carbon black, a form of carbon used as a filler in tires.[42] For the plastics subsector, the most important inputs are raw forms of different common plastics — polyethylene (PET), polypropylene (PP), polyvinyl chloride (PVC), polystyrene, and others — often purchased in the form of pellets. Although the pellets may vary in their chemical properties, these differences are typically noted on the packaging. Within a given chemical specification, pellets from different producers and origin countries are typically considered to be highly substitutable. There may be other dimensions of supply relationships that cannot be observed ex ante, for instance timeliness of delivery or willingness of supplier to extend trade credit. But to a first approximation we believe it is reasonable to treat the main inputs in rubber and plastics as highly substitutable, with observable quality differences.

As is evident in Table 2, a large share of inputs in both subsectors is imported. In rubber products, almost all natural latex is imported, as is a substantial share of carbon black and other inputs. In plastics, a majority of PET and 20-25% of PVC and polystyrene are imported. These import shares are from the EAM data and hence represent shares of inputs imported directly by firms. To the extent that firms purchase imported goods from local intermediaries, they understate the true import shares of the inputs. Both rubber and plastics are among the more import-intensive 3-digit subsectors in the EAM data.

In short, because both rubber and plastics products producers use highly substitutable inputs, a large share of which are imported, to produce horizontally and vertically differentiated products, we believe that they are well-suited to our method.

In selecting the estimation sample, we require that a firm have complete data on capital,

[42] The 5-digit product code 32431, harmonized at the international level, includes "inorganic oxygen compounds of boron, silicon and carbon." The final two digits of the CPC product code are Colombia-specific. We have confirmed in the firm-level imports data (using an alternative 8-digit product classification) that the main input in this category is carbon black and other forms of carbon.

labor, materials, and outputs for at least six consecutive years. This requirement is helpful to ensure that the perpetual-inventory method generates a sensible measure of capital stock. It also ensures that our sample of firms does not change as we modify the number of lags required in different specifications. Given that the EAM data are available beginning in 1994 and that in all specifications we will need at least two lags, our sample covers the years 1996-2009. We are left with 362 firms in an unbalanced panel, with 11.73 observations per firm on average. Table 3 presents summary statistics on this baseline sample. We see that the two subsectors are comparable on many dimensions. Rubber firms spend a large share of expenditures on imports and earn a larger share of revenues from exports, but employment averages about 100 in both subsectors and wages and output are similar.

To explore the robustness of our results to the definition of the subsectors of interest, in the appendix we will report results for two alternative samples. In the first, we remove rubber products and focus exclusively on plastics, the larger of the two subsectors. In the second, we keep both rubber and plastics and add glass products (ISIC rev. 2 subsector 362), a subsector that also arguably satisfies the criteria of relatively substitutable inputs, high imported input share, and differentiated outputs. Descriptive statistics on the glass products subsector are reported in Appendix Tables A1-A2.

4 Baseline Results

This section reports the results of the estimation strategy laid out in Section 2. For comparison purposes, we begin by presenting the "naive" OLS and first-difference (FD) results, and then move on to our two-step IV (TSIV) method.

4.1 "Naive" OLS and FD Models

Panel A of Table 4 presents estimates using sales as the measure of output and material expenditures as the measure of input use, with both deflated by sector-level deflators. The OLS estimates in Columns 1 and 2, without and with year effects respectively, appear to be reasonable, and are roughly consistent with constant returns to scale, as is typically expected (see e.g. Bartelsman and Doms (2000)). Columns 3 and 4 report first-difference (FD) estimates, corresponding to equation (16) without instruments. Relative to the OLS estimates, the materials coefficients are significantly lower, the labor coefficients remain roughly unchanged, and, strikingly, the capital coefficients drop almost to zero. The latter fact illustrates a point made above, that transformations to remove firm effects lead to severe attenuation of the capital coefficient; this problem is not specific to our TSIV method.

Panel B of Table 4 again report OLS and FD estimates, but using the Sato-Vartia quantity aggregates for output and materials. In Columns 1 and 2, we have imposed the firm-specific normalizations for $\widetilde{y}_{i0}$ and $\widetilde{m}_{i0}$ discussed in Section 2.4.2 above, effectively using each firm's first year in the unbalanced panel as the base year for the firm-specific output and input deflators. Overall, we see significant differences in the OLS estimates — in particular, deflating at the firm level reduces the OLS materials coefficient and raises the capital coefficient — but the FD estimates in Columns 3 and 4 are quite similar to those using sales and expenditures in Panel A.

4.2 Differences (Step 1) Results

In this step, we estimate our difference equation (16) using instruments for the changes in input choices. Table 5 reports the first stage for different sets of instruments. Columns 1-3 use only internal instruments, and in particular only lagged levels of inputs from period $t-2$. The coefficient estimates are plausible, with lagged levels negatively associated with current changes. But the instruments are weak: the Sanderson and Windmeijer (2016) (SW) F-statistics are below the rule-of-thumb level of 10 (as are the conventional F-statistics for materials and labor), and although the Kleibergen-Paap (KP) LM test easily rejects the null of underidentification, the KP Wald statistic for weak instruments is below 1. In Appendix Table A3, we show that this weak instrument issue is not resolved by including further lags as instruments in a GMM model.[43]

To improve the explanatory power of the first stage of this step, we turn instead to our external instruments. As described in Subsection 2.4.1 above, the first step in the construction of the firm-level predicted import-price index, $\triangle\widehat{\widetilde{w}}_{it}^{imp}$ from (20), is to estimate the relationship between RER movements and import prices given by (19), leaving out the data from one firm at a time. (In running these regressions, we include all firms in the DIAN imports data that can

[43] Appendix Table A3 reports results from GMM estimation of our difference equation (16), where further lags have been added "GMM-style" (Holtz-Eakin et al., 1988; Roodman, 2009), using available lags and allowing separate coefficients in each period. Lags are included just to $t-2$ in Column 1, to $t-3$ in Column 2, and to the firm's initial year in Column 3. The Kleibergen-Paap Wald statistic remains below 2 and the Sanderson-Windmeijer F statistics are all below 3.5. However, the materials and labor coefficient estimates are similar to those using our TSIV procedure presented below.

be linked to the EAM manufacturing data, not just those in the rubber and plastics subsectors.) This generates 362 sets of coefficient estimates, one for each firm in our sample, where each set contains a separate coefficient for each sector-year. In Appendix Figure A4, we report sector-level average coefficients, averaging across firms and across years (with standard errors also averaged across firms and years). Although there is some heterogeneity, in the majority of sectors (and on average across sectors) import prices are positively related to RER movements, as expected.

Columns 4-6 of Table 5 report the first-stage estimates including the two external instruments — the predicted change in import price, $\triangle\widehat{\overline{w}}_{int}^{imp}$ from (20), and the predicted wage change instrument, $\triangle z_{it}$ from (22) — and one internal instrument, the lagged level of capital from $t-2$. The coefficient estimates broadly conform to our expectations. In particular, the predicted import price change is significantly negatively related to the change in the material quantity aggregate and the predicted wage change is significantly negatively related to the change in employment. In the latter case, the predicted wage change is also negatively related to the materials and capital changes. The instruments are somewhat stronger than in the internal-instruments-only model in Columns 1-3, but both the SW F-statistic for materials and labor and the KP Wald statistic continue to warrant concern about the weakness of the instruments.

Our preferred specification combines the three internal instruments from $t-2$ and the external instruments. The corresponding first stage is reported in Columns 7-9 of Table 5. The coefficient estimates are similar to those in the other columns but the strength of the instrument set has improved. The SW F-statistic is above the rule-of-thumb level of 10 for labor and capital and the KP Wald statistic, while still somewhat low at 3.363, is noticeably larger than in the other columns. The concern about the weakness of instruments remains, but it has been mitigated by the inclusion of the external instruments.

Table 6 presents the second-stage estimates for the three instrument sets in Table 5. In the first two columns, the coefficients on materials and labor are imprecisely estimated and change markedly across columns, as one might expect given the weakness of the instruments in these specifications. In our preferred specification in Column 3, by contrast, the materials and labor coefficients are more precisely estimated and are of plausible magnitudes, .38 and .40 respectively. The labor coefficient is substantially larger than, and the materials coefficient very similar to, the corresponding FD estimates in Table 4, Panel B, Columns 3-4. The difference in the labor coefficient is consistent with the presence of an output-quality bias discussed above: if producing higher-quality output requires more labor, then we would expect a positive correlation between $\triangle\ell_{it}$ and $\triangle q^y_{it}$ in (16), generating a negative bias in OLS and FD estimates of β_ℓ, which our approach would correct.[44]

As previewed above, the capital coefficient is implausibly low in this specification. The point estimate is in fact negative, although the confidence interval allows for positive values of roughly the magnitude of the OLS estimate in Columns 3-4 of Table 4, Panel A. We believe that the reason for the erratic estimate is that most of the genuine variation in capital stock is between rather than within firms, and that once first-differencing removes the firm effect, much of the remaining within-firm variation is due to measurement error. In Step 2 below, using the levels equation, we will use the variation between firms to arrive at a more plausible point estimate for the capital coefficient.

Given that the SW F-statistic for materials and the KP Wald statistic are still somewhat low in our preferred specification, we explore the robustness of the estimates in two ways. First, we report weak-instrument-robust confidence intervals. The econometric literature has not reached consensus on the best method for estimating these intervals, especially in the non-homoskedastic case. Here we follow the approach of Andrews (2016, 2018), which uses a statistic based on a linear combination (LC) of the K statistic of Kleibergen (2005) and the S statistic of Stock and Wright (2000).[45] We treat β_k as a nuisance parameter (without assuming it is strongly identified); the confidence intervals for β_m and β_ℓ are calculated using a projection method due to Chaudhuri and Zivot (2011). These intervals are reported in Column 3 of Table 5. The intervals are centered at the reported point estimates and allow us to reject the nulls that $\beta_m = 0$ and $\beta_\ell = 0$ comfortably at the 95% level. Second, to further probe robustness, we estimate the Column 3 specification using limited-information maximum likelihood (LIML), which has been found to be more robust to weak instruments than IV (Stock et al., 2002; Angrist and Pischke, 2009). The Andrews LC robust confidence intervals, reported in Column 4, are somewhat larger, but the coefficient estimates are nearly identical to those in Column 3, which is reassuring. While it would be preferable to have stronger instruments, we interpret these results as indicating that our first-step IV estimates of

[44]Another possibility is that measurement error in labor generated attenuation bias in the FD estimate, which our procedure removes.

[45]In practice, the confidence region is found by using this linear combination (referred to as the LC statistic) to test the null hypothesis at each point on a grid roughly spanning the parameter space; the confidence region is then the set of points at which the null cannot be rejected. The LC statistic provides more powerful tests than the K or S statistics alone under some circumstances. Conveniently, this procedure is implemented in Stata by the twostepweakiv command (Sun, 2018).

β_m and β_ℓ are robust to weak-instrument concerns.

4.3 Levels (Step 2) Results

We now turn to the second step of our TSIV procedure. We consider the levels equation, (24), where we plug in the difference equation (Step 1) estimates, $\widehat{\beta}_m$ and $\widehat{\beta}_\ell$, on the left-hand side.

Panel A of Table 7 reports the first stage of the IV procedure for this step. Our preferred specification, using $\triangle k_{it-1}$ as the instrument for k_{it} appears in Column 1. To check robustness, we also report results using $\triangle k_{it-2}$ as the instrument in Column 2. In neither case is there a weak-instrument concern: the Kleibergen-Papp Wald statistic is above 36 in both cases.[46] Although the R-squared of the first-stage regression is low, the first-stage coefficient is .67 in both columns and highly significant.

Panel B of Table 7 reports the corresponding second stage. Columns 1 and 2 correspond to the same Columns in Panel A, with $\triangle k_{it-1}$ and $\triangle k_{it-2}$ as instruments, respectively. For comparison purposes, Column 3 reports the OLS estimate, without instrumenting k_{it}. Below each estimate, we report the uncorrected standard errors in parentheses and the corrected standard errors in brackets. As noted in Subsection 2.4.2, the uncorrected standard errors are appropriate in the special case that the instrument is uncorrelated with $\widetilde{m}_{it}^{SV}$ and ℓ_{it}. Otherwise, the corrected standard errors are the appropriate ones. It is important to acknowledge that with the corrected standard errors, the confidence intervals in both columns are wide and we cannot reject the null that $\beta_k = 0$ at conventional levels.

Despite the large standard errors following the correction, we consider these results to be reassuring. The point estimate in our preferred (Column 1) specification, .20, is plausible, and (together with the first-step estimates of $\widehat{\beta}_m$ and $\widehat{\beta}_\ell$, .38 and .40) indicates that that returns to scale are very nearly constant, as is generally expected (Bartelsman and Doms, 2000).[47] While one would of course prefer to have a more precise estimate of β_k, we have more confidence in the estimate than in the close-to-zero estimates from the naive first-differences models in Columns 3-4 of Panels A and B of Table 4 or the negative estimates from the Step 1 difference equation in Table 6.

[46] The Kleibergen-Papp Wald statistic and the Sanderson-Windmeijer F statistic coincide in cases with a single endogenous covariate.

[47] If we instead use the Column 2 estimate of .29, the materials, labor and capital coefficients sum to 1.07 but this is not statistically significantly different from 1.

It is notable that our TSIV estimates of .38, .40, and .20 for β_m, β_ℓ, and β_k are, in the end, quite similar to (and not statistically different from) the naive OLS results using our input and output quantity aggregates in Columns 1-2 of Table 4, Panel B. One might reasonably ask what has been gained by going through our new procedure. We note that it was by no means obvious at the beginning that the TSIV estimates would be so similar to the OLS-in-levels estimates. The labor and capital coefficients drop after first-differencing to remove the firm effect, and then rise again with our procedure. Although many explanations are possible, this pattern is consistent with the hypothesis that there are two offsetting biases in the naive OLS results: a transmission bias due to the firm effect, η_i, and a quality bias due to greater use of labor for higher-quality outputs. While these two biases may offset in the current setting, they may not always, and it is potentially quite useful to have a method that can address both. Comparing to the naive OLS estimates using sales and purchases in Columns 1-2 of Table 4, the fact that the materials coefficient is significantly smaller when using the quantity aggregates, both in the OLS estimates in Columns 5-6 of Table 4 and our TSIV estimates, underlines the importance of having access to quantity information on inputs and outputs.

5 Robustness

5.1 Alternative Aggregators

Our within-firm CES assumptions are convenient for showing theoretically how quality and variety differences may bias estimates of output elasticities, but they are admittedly restrictive. It is natural to ask whether our particular functional-form assumptions are driving our results. It turns out that the empirical patterns are robust to using other common aggregators to aggregate from the firm-product to the firm level: a Tornqvist index, a Paasche index, and a Laspeyres index. Appendix Tables A4-A6 report estimates analogous to Tables 5-7 using these alternative aggregators and the combination of external and internal instruments. The results are qualitatively similar. The point estimates display small differences from our baseline estimates — the Tornqvist materials coefficient is larger, the Paasche labor coefficient is smaller, and the Laspeyres capital coefficient is smaller — but these differences are not statistically significant. Using the weak-instrument-robust confidence intervals, we see that the Laspeyres materials coefficient is not significantly different from zero at the 95% level, but it is significant at the 90% level. Overall, these results suggest that the empirical patterns are not particularly sensitive to the functional form of the aggregator that we use.

5.2 Adding Predicted Export Price Index

Changes in real exchange rates in export destinations may affect export prices and how much firms sell in those destinations, in addition to the prices of imports. If export destinations are correlated with import origins at the market level and individual-firm level in Colombia, that could generate a correlation between our predicted import price index, $\triangle\widehat{\overline{w}}_{it}^{imp}$, and the error term, $\triangle u_{it}$. To address this concern, we construct a predicted *export* price index, $\triangle\widehat{\overline{w}}_{it}^{exp}$, analogous to the predicted import price index, and include it as a covariate. We follow the same steps as for constructing the predicted import price index: we generate leave-one-out estimates of export price changes and then average these using the composition of firms' export baskets. In Appendix Tables A7-A8, this predicted export price index is included as an additional covariate. The results are very similar to our baseline estimates. The 95% weak-instrument-robust confidence interval for the materials coefficient includes zero, but we can still reject the null that $\beta_m = 0$ at 90% confidence. The basic patterns are robust to including the export price control.

5.3 Alternative Samples

As discussed in Subsection 3.4, in choosing which subsectors to focus on we have faced a trade-off between increasing sample size and reducing cross-firm heterogeneity. To explore this trade-off further, we present estimates for two additional samples. In the first, we include only producers of plastic products (ISIC rev. 2 code 356). In the second, we add another subsector that also, arguably, uses relatively homogeneous inputs, produces differentiated outputs, and imports a substantial share of inputs: producers of glass products (ISIC rev. 2 code 362).[48] Appendix Tables A9-A11 report the estimates for the alternative samples. Unsurprisingly, the precision of the estimates is increasing in sample size. The weak-instrument statistics signal greater reason for concern in the plastics-only sample, and somewhat less reason in the combined rubber, plastics, and glass sample. But overall, the patterns are similar to the sample with rubber and plastics producers in our baseline estimates.

[48] Recall that Appendix Tables A1-A2 report summary statistics for glass products.

6 Comparison to Output Elasticities from Other Methods

This section compares our output-elasticity estimates to those of other commonly used methods applied in our sample, using log sales and log expenditures rather than our Sato-Vartia quantity indexes. As noted above, our approach is perhaps closest in spirit to System GMM (Arellano and Bover, 1995; Blundell and Bond, 1998, 2000). The standard System GMM set-up has a Cobb-Douglas production function and a firm effect, as we do, but it assumes the ex ante component of productivity (here ω_{it}) follows an AR(1) process. After quasi-differencing to remove the serial correlation in the error, the main estimating equation contains a lagged dependent variable as well as lags of covariates. Difference and levels equations are then estimated simultaneously.

Table 8 presents the results from applying standard System GMM in our baseline sample. We include time fixed effects and use the "two-step" procedure described in Roodman (2009), using the initial weighting matrix defined in Doornik et al. (2012) and implementing the Windmeijer (2005) finite-sample correction for the resulting covariance matrix.[49] The coefficients on contemporaneous log expenditures, log labor, and log capital are estimates of the Cobb-Douglas output elasticities, corresponding to our β_m, β_ℓ, and β_k.[50] The columns differ in the number of lags of the covariates in levels that are included in the difference equation, with lags just from $t-2$ in Column 1, from $t-2$ and $t-3$ in Column 2, and from all available periods starting from $t-2$ in Column 3. The instruments are included "GMM-style," effectively interacted with year dummies (Holtz-Eakin et al., 1988; Roodman, 2009). In the corresponding levels equations, we include the first lags of the first-differenced covariates as instruments.[51] To gauge the strength of the instruments, we follow Bazzi and Clemens (2013) and Kraay (2015) in reporting weak-instrument diagnostics separately for the differences and levels equations in Appendix Table A12. In the differences equation, the number of instruments the rises quickly (given that lags from different years are considered different instruments), from 56 in Column 1 to 420 in Column 3. The set of instruments appears to be weak, with the Kleibergen-Paap Wald test statistic below 2 and the Sanderson-Windmeijer F-statistics below 2 for contemporaneous labor and capital, and below 3

[49] In particular, we use the Stata xtabond2 command of Roodman (2009) with options h(2), twostep, and robust. Following Roodman's replication of Blundell and Bond (1998), we include time fixed effects as instruments only in the levels equation, since they are asymptotically redundant in the difference equation.

[50] The model implies additional restrictions on the relationship between the coefficients on the contemporaneous and lagged terms, which we do not test here.

[51] Further lags of differences in the levels equation are redundant (Arellano and Bover, 1995; Blundell and Bond, 2000).

for contemporaneous materials expenditures.[52] That said, it is worth noting that the resulting coefficient estimates are not very far from our baseline estimates: focusing on the specification in Column 3 (using all available instruments), the point estimate of .48 for materials is larger than our $\widehat{\beta}_m^{TSIV}$ of .38, the point estimate of .26 for labor is smaller than our $\widehat{\beta}_l^{TSIV}$ of .40, but these differences are not statistically significant. The point estimate for capital of .03 is significantly smaller than our $\widehat{\beta}_k^{TSIV}$ of .20.[53]

Table 9 reports estimates from four leading proxy-variable methods: those of Olley and Pakes (1996) (OP), Levinsohn and Petrin (2003) (LP), Wooldridge (2009) (using materials as the proxy), and Gandhi, Navarro and Rivers (2020) (GNR).[54] The method of Ackerberg, Caves and Frazer (2015) (ACF) is also commonly used, but the authors recommend that it only be used with value-added production functions, not gross output functions, and hence their coefficient estimates are not directly comparable to ours. For OP, LP and Wooldridge in Columns 1-3, the point estimates for materials are consistently higher than our baseline estimate, and fall outside our weak-instrument-robust confidence interval.[55] The point estimate for labor is consistently lower than our baseline estimate, although not significantly so. The GNR estimate of the materials coefficient in Column 4 is very similar to ours, and the labor coefficient is somewhat larger although not significantly so.[56] Overall, although the differences in estimates are generally not statistically significant, it is worth emphasizing that they will be of consequence when the point estimates are used to estimate markups or productivity, as we will see in the next section.

7 Measures of Productivity

Our main objective in this paper is to estimate the output elasticities consistently, without price, quality, or variety biases, and we have argued that our method achieves this goal. But once we

[52] The Hansen test of over-identifying restrictions is appropriate in the non-homoskedastic case and does not reject the hypothesis that the instruments are jointly valid. But it should be interpreted with caution, as it is weakened by the presence of many instruments (Roodman, 2009).

[53] In Appendix Table A13, we report the results of System GMM estimation with our quantity aggregates in place of revenues and expenditures. The results are broadly similar, with slighly lower estimates of the materials and labor elasticities, and somewhat higher estimate of the capital elasticity.

[54] For OP, LP and Wooldridge, we use the Stata command prodest (Rovigatti and Mollisi, 2018); for GNR, we have coded the estimation ourselves. For all specifications we obtain standard errors by using a bootstrap with 50 replications.

[55] Note that for OP a large number of firms report zero investment in capital for some years and are dropped from the estimation.

[56] In implementing GNR, given the Cobb-Douglas structure of the the production function, we use a polynomial of degree zero for the materials expenditure elasticity, a polynomial of degree one in capital and labor for the integration constant, and a polynomial of degree three for the AR(1) process of ω_{it} (i.e. $\omega_{it} = \sum_{a=1}^{3} \delta_a \omega_{it-1}^a$).

have the elasticity estimates in hand, it is natural to ask whether we can use them to construct a measure of productivity. In this section, we define and discuss productivity measures using our estimates (Subsection 7.1) and compare our preferred measure to other common methods in predicting export performance (Subsection 7.2).

7.1 Definitions

Our output-elasticity estimates can be used to construct two different TFP measures, depending on whether we use our Sato-Vartia quantity indexes or revenues and expenditures. We refer to these two measures as $TFPQ'$ and $TFPR'$. Neither measure captures only technical efficiency, but their shortcomings differ, in ways that are useful to examine. In this subsection we focus on changes in productivity rather than levels, because in changes we can avoid taking a stand on the firm-specific normalizations, $\widetilde{y}_{i0}$ and $\widetilde{m}_{i0}$.

To define $\triangle TFPQ'$, we use the observable Sato-Vartia quantity indexes, $\widetilde{y}_{it}^{SV}$ and $\widetilde{m}_{it}^{SV}$:[57]

$$\triangle TFPQ'_{it} = \triangle \widetilde{y}_{it}^{SV} - \widehat{\beta}_m \triangle \widetilde{m}_{it}^{SV} - \widehat{\beta}_k \triangle k_{it} - \widehat{\beta}_\ell \triangle \ell_{it} \tag{25}$$

In our model, technical efficiency is represented by $\triangle\omega_{it} + \triangle\epsilon_{it} + \triangle\xi_t$.[58] But referring to (16), we see that:

$$\operatorname*{plim}_{I\to\infty} \triangle TFPQ'_{it} = \triangle\xi_t + \triangle\omega_{it} + \triangle\epsilon_{it} + (\beta_m \triangle v_{it}^m - \triangle v_{it}^y) + (\beta_m \triangle q_{it}^m - \triangle q_{it}^y) \tag{26}$$

That is, changes in quality and variety of both outputs and inputs are captured in $\triangle TFPQ'$. If we can can be confident that output and input quality and variety are roughly constant over time — as for instance for single-product, single-input firms in homogeneous-good industries — then $\triangle TFPQ'$ is a consistent estimator for technical efficiency. But to the extent that a firm increases output quality or variety, $\triangle TFPQ'$ will understate increases in technical efficiency, and to the extent that it increases input quality or variety, it will overstate them. This may explain, for instance, why the Egyptian rug producers randomly allocated an initial export contract in Atkin et al. (2017, 2019) saw increases in output quality, profitability, and productivity measured under

[57]Note that in the case of single-product, single-input firms, $\widetilde{y}_{it}^{SV}$ and $\widetilde{m}_{it}^{SV}$ reduce to the physical quantity of the single output and input.

[58]There is some difference in practice in whether to include the year effect, $\triangle\xi_t$, in the definition of TFP. Here we do, but note that it can be removed by deviating from year means.

laboratory conditions, but a decrease in physical-quantity-based TFP.

To define $\triangle TFPR'$, we use firm revenues and expenditures, r_{it} and e_{it}, in place of the output and input quantity indexes, $\widetilde{y}_{it}^{SV}$ and $\widetilde{m}_{it}^{SV}$:

$$\triangle TFPR'_{it} = \triangle r_{it} - \widehat{\beta}_m \triangle e_{it} - \widehat{\beta}_k \triangle k_{it} - \widehat{\beta}_\ell \triangle \ell_{it} \tag{27}$$

Given $r_{it} = \widetilde{y}_{it} + \widetilde{p}_{it}$, $e_{it} = \widetilde{m}_{it} + \widetilde{w}_{it}$, and the relationships in (13)-(14), it follows that:

$$\operatorname*{plim}_{I\to\infty} \triangle TFPR'_{it} = \triangle\xi_t + \triangle\omega_{it} + \triangle\epsilon_{it} + \triangle\widetilde{p}_{it} - \beta_m \triangle \widetilde{w}_{it}$$

Relative to $\triangle TFPQ'$, $\triangle TFPR'$ has the advantage that changes in quality and variety are absorbed in the revenues and expenditure terms. But it has the disadvantage that it captures pure price changes, reflected in the price index changes, $\triangle\widetilde{p}_{it}$ and $\triangle\widetilde{w}_{it}$. Abstracting from variety changes, if there are no changes in quality-adjusted prices (e.g. if both quality and prices are constant, or if price changes fully reflect quality changes and not other factors), then $\triangle\widetilde{p}_{it} = \triangle\widetilde{w}_{it} = 0$ and $\triangle TFPR'$ reflects changes in technical efficiency only. On the other hand, if quality-adjusted prices of outputs or inputs change, they show up in $\triangle TFPR'$.

Whether $\triangle TFPQ'$ or $\triangle TFPR'$ is the more appropriate measure thus depends to some extent on the setting and analytical objective, with $\triangle TFPQ'$ better suited in homogeneous industries and $\triangle TFPR'$ arguably more informative in industries with greater vertical differentiation and/or output and input specialization. In the case of the rubber and plastics industries in Colombia, in which quality differences are likely to be important, we believe that $\triangle TFPR'$ is the more appropriate measure. At the same time, it is important to remain aware that it may reflect price changes as well as changes in technical efficiency.

7.2 Comparison with Other TFP Measures

How does our preferred measure of productivity compare to other standard measures? As a first illustration, Appendix Table A14 reports pairwise correlation coefficients between the levels of TFP calculated using different methods.[59] To the methods in Tables 8, 9 and A13 mentioned above, we add estimates from the Ackerberg et al. (2015) method, using a value-added production

[59] In levels, we define:

$$TFPR' = r_{it} - \widehat{\beta}_m e_{it} - \widehat{\beta}_k k_{it} - \widehat{\beta}_\ell \ell_{it} \tag{28}$$

function, and from the OLS estimation with quantity aggregates from Table 4. Overall, the different measures are reasonably highly correlated with one another, an observation that has been made for instance by Van Biesebroeck (2008) and Eslava et al. (2009) for different sets of methods, but the correlations are far from perfect. Our measure is most highly correlated with the System GMM estimates, LP and Wooldridge, and least correlated with OP and ACF.

One way to begin to evaluate the performance of different estimators is to examine how well they predict outcomes that one can be fairly confident are related to firms' technical efficiency. Here, as a first step, we relate estimated changes in productivity to future changes in export performance. Although there is little agreement in the literature about whether exporting itself increases productivity, there is a broader consensus that productivity increases are likely to predict future expansions of exports (Bernard and Jensen, 1999; Alvarez and López, 2005; Costantini and Melitz, 2008; Aw et al., 2011). Relating productivity to future export growth also avoids concerns about how changes in markups or product composition due to exporting might affect contemporaneous productivity estimates.

Table 10 reports correlations between changes in the various productivity measures from period $t-s$ to t and changes in the inverse hyperbolic sine of the value of exports from t to $t+s$, where $s \in \{1,2,3\}$. We standardize all measures so that they have standard deviation 1 (pooling years) and we include year effects.[60] In all specifications, the R^2 is quite small, and for the single-period differences ($s=1$), none of the TFP measures are significant predictors of future exports. But for longer differences, we find that all of the productivity measures are significant predictors of future export growth. Given that the measures have been standardized, the magnitude of the coefficients can be interpreted as an indicator of the strength of the relationship, and we see that our measure is a stronger predictor of future export growth than the other measures we have considered. Although the explanatory power of all of the measures is quite low, our measure has the largest R^2 among the set. These results are by no means definitive, but we view them as suggestive that our measure represents a modest improvement in measuring technical efficiency in our setting.

using the notation from above, where r_{it} and e_{it} is material expenditures. For the other methods, TFP is defined analogously, using log revenues.

[60] Note that the longer the differences we use, the fewer observations we are able to include in the sample, since we lose observations either at the beginning or the end of the spell in the sample for each firm.

8 Conclusion

This paper has developed a method for estimating production-function parameters that can be applied in differentiated-product industries. We believe that three contributions stand out. First, we have highlighted the pitfalls of using physical quantities to estimate output elasticities or TFP in industries where quality and variety vary by firm and over time. Using within-firm CES aggregators for outputs and material inputs, we have shown theoretically how standard estimates of output elasticities are likely to be biased by quality and variety differences. Second, we have used external drivers of input prices as instruments to estimate the elasticities of output with respect to materials and labor. The idea of using external instruments is not new (Griliches and Mairesse, 1998), but previous authors have not had access to the combination of datasets and naturally occurring variation that we have been able to take advantage of here. Third, we have developed an approach to improving estimates of the output elasticity with respect to capital. It is well known that models in first-differences yield unsatisfactory estimates of the capital coefficient, and our first-step model is not an exception. In the spirit of System GMM, we have added a levels equation, using a lagged difference as an instrument. Our two-step IV (TSIV) approach has the advantage that the estimates of the materials and labor coefficients are robust to misspecification of the second step, which requires stronger assumptions. Our method addresses the quality and variety biases we have identified, in addition to transmission bias, and measures of productivity changes constructed using our estimates do well in comparison to other standard measures in predicting future export growth at the firm level.

Two important questions remain open. The first is what to do if one does not have the combination of rich data and naturally occurring variation that we have in our setting. Data on physical quantities of both inputs and outputs are becoming increasingly available, including in Spain (Doraszelski and Jaumandreu, 2013, 2018), Portugal (Bastos et al., 2018), the United States (Roberts and Supina, 1996; Foster et al., 2008; Atalay, 2014), Chile (Garcia-Marin and Voigtländer, 2019), Ecuador (Bas and Paunov, 2020), Bangladesh (Cajal Grossi et al., 2019), Japan in the Meiji period (Braguinsky et al., 2015), China (Brandt et al., forthcoming) and India (Gupta, 2020). But it may be difficult to find credible external instruments and internal instruments may be weak. One potential way forward would be to construct proxies for the quality and variety terms that appear in estimating equations such as (14)-(15) above. The approach of De Loecker et al. (2016) of including a flexible function of output price and market share on the right-hand-side is a promising step in this direction. One could also consider constructing explicit measures of quality, as for instance Khandelwal et al. (2013) do for output quality; such a strategy would require imposing more structure on consumer demand than we have been willing to do here, but may be warranted in some circumstances. To proxy for variety, one could include the observable components of the variety terms derived above and allow for firm-specific coefficients on them.[61] It is also worth noting that in our setting, OLS in levels using the firm-level output and input quantity aggregates yield estimates very similar to our TSIV procedure; the biases due to unobserved quality and variety changes and the biases deriving from correlations between input choices and time-invariant firm effects appear to offset. More research is required to determine if this generalizes to other contexts; if it does, such OLS estimates may prove to be very useful measures in settings where quantity data are available but credible external instruments are not.

The second big open question is how to arrive at a measure of changes in technical efficiency that excludes both pure price changes and changes in unobserved quality and variety. We believe that our method yields an improved set of output elasticity estimates. But even with these improved estimates, neither quantity-based TFP nor revenue-based TFP is an ideal measure of technical efficiency, as discussed above in Section 7. The proxy strategies mentioned in the previous paragraph represent one way forward, as they would provide direct measures of the quality and variety terms that could be used to correct quantity-based TFP. Another approach would be to use revenue-based TFP but to estimate markups explicitly in order to purge them from the productivity estimate. Garcia-Marin and Voigtländer (2019) and Blum et al. (2018) have recently pursued a strategy in this spirit, using the method of De Loecker et al. (2016) to estimate markups. We believe that our method for estimating output elasticities will help to improve estimates of markups in the approaches of De Loecker and Warzynski (2012) and De Loecker et al. (2016) and hence could help to improve revenue-based TFP measures along these lines.

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Table 1. Primary Outputs, Rubber and Plastic Products Producers

CPC code	Share of total revenues	CPC description of outputs
A. Rubber Products Producers		
3611301	0.52	Rubber tires, of a kind used on buses and trucks
3611101	0.16	Rubber tires, of a kind used on automobiles
3612001	0.07	Retreaded pneumatic tires
3611303	0.05	Rubber tires, of a kind used on agricultural vehicles and machines
3626001	0.03	Rubber gloves
3627217	0.02	Rubber separators for batteries
3611502	0.02	Strips for retreading rubber tires
3611405	0.02	Pneumatic tires, of a kind used on buses and trucks
3627220	0.02	Rubber cushions
3624002	0.02	Rubber conveyor belts
3611501	0.01	Camel-back strips for retreading tires
3627216	0.01	Rubber spare parts for automotive and machinery
3627218	0.01	Printing blankets
3626004	0.01	Surgical gloves
3611401	0.01	Rubber protectors for tires
3791010	0.01	Abrasive cloths and fabrics for cleaning
3611404	0.01	Pneumatic tires, of a kind used on motor car
3542009	0.00	Rubber-based adhesive
3627207	0.00	Rubber articles for electrical use
3622202	0.00	Rubber mixtures n.e.c. (not elsewhere classified)
B. Plastic Products Producers		
3632001	0.09	Polyvinyl tubing
3641006	0.08	Printed plastic bags
3641003	0.08	Printed plastic film in tubular form
3633011	0.07	Polypropylene film
3633004	0.07	Polyethylene film
3641004	0.06	Unprinted plastic bags
3649007	0.06	Plastic caps and lids
3633012	0.05	Plastic laminated film
3649014	0.05	Blister packaging for medicines
3649002	0.05	Plastic containers of a capacity not exceeding 1000 cm3
3649003	0.05	Plastic containers of a capacity exceeding 1000 cm3
3633014	0.04	Printed polyethylene film
3694013	0.04	Plastic straws
3633008	0.04	Acrylic sheets
3641005	0.04	Synthetic sacks
3632008	0.03	Fabrics of polypropylen in tubular form
3649008	0.03	Plastic container for drugs and medicines
3633007	0.02	Polyvinyl film
3639201	0.02	Polyvinyl film with textile material
4153504	0.02	Laminated aluminum foil

Notes: Baseline sample, producers of rubber and plastic products (ISIC rev. 2 categories 355 and 356), 1996-2009. Shares for each industry calculated as revenues from output over total revenues for 2000-2009 period, pooling firms and years.

Table 2. Primary Inputs, Rubber and Plastic Products Producers

CPC code	Share of total expenditures	CPC description of inputs
A. Rubber Products Producers		
0321001	0.40	Natural latex
3423112	0.12	Carbon blacks and other forms of carbon
2799601	0.07	Tire cord fabric
2819004	0.05	Fabric of synthetic fiber in tubular form
3611502	0.05	Strips for retreading rubber tires
4126301	0.04	Iron or steel cable
3478007	0.04	Nylon
3549405	0.03	Stabilizers for synthetic resins
0321002	0.03	Natural rubber in primary forms or in plates, sheets or strips
3633021	0.03	Polypropylene fabric
3549934	0.03	Food emulsifier
3480301	0.02	Synthetic latex
3622101	0.02	Rubber sheets
3549403	0.02	Vulcanization Accelerators
3543102	0.01	Mineral oils
3422101	0.01	Dioxide, zinc oxide
3549401	0.01	Plasticizers
3611402	0.01	White strips for tires
3474002	0.01	Polyester resins
3926001	0.01	Used tires
B. Plastic Products Producers		
3471001	0.35	Polyethylene
3476001	0.14	Polypropylene
3473002	0.12	Polyvinyl chloride
3472001	0.05	Polystyrene
3479902	0.04	Synthetic emulsions
3474002	0.04	Polyester resins
3549404	0.03	Plastics additive
3513004	0.03	Alcohol-based flexographic inks
3633011	0.03	Polypropylene film
3434007	0.03	Colorants for plastics
3415901	0.02	Diisocyanates - desmophens - desmodurs
3411403	0.02	Styrene
3215302	0.02	Corrugated cardboard boxes
3633004	0.02	Polyethylene film
3477002	0.02	Acrylic resins
3479903	0.02	Homopolymers
3474007	0.01	Polyacetal thermoplastic resin
3549401	0.01	Plasticizers
3413307	0.01	Polyols
4153501	0.01	Aluminum foil

Notes: Baseline sample, producers of rubber and plastics products. Shares for each industry calculated as expenditures on input over total expenditures for 2000-2009 period, pooling firms and years.

Table 3. Summary Statistics

	Rubber	Plastics	All
A. Period: 1996-2009			
Number of Observations	554	3693	4247
Number of Firms	46	316	362
Number of Workers	98.08	101.37	100.94
Production value (billions 2000 pesos)	10.46	8.83	9.04
Earnings per year, permanent workers (millions 2000 pesos)	7.14	7.00	7.02
B. Period: 2000-2009			
Input variables			
No. inputs per firm in average firm-year	11.66	8.01	8.46
Share of firms that import	0.61	0.59	0.60
No. inputs per firm in avg. firm-year, cond. on importing	16.57	10.34	11.18
Fraction of expenditure on imported inputs	0.23	0.18	0.19
No. imported HS8 categories in avg. firm-year, cond. on importing	29.55	19.41	20.47
Output variables			
No. outputs per firm in average firm-year	3.54	3.08	3.13
Share of firms that export	0.48	0.55	0.54
No. outputs per firm in avg. firm-year, cond. on exporting	5.26	3.83	4.00
Fraction of revenues from exported outputs	0.08	0.06	0.06
No. exported HS8 categories in avg. firm-year, cond. on exporting	5.64	5.38	5.41

Notes: Baseline sample, rubber and plastics products producers, 1996-2009. Table reports averages of firm-level values (giving every firm equal weight). Exports and imports available in EAM data only in 2000-2009. Average 2000 exchange rate is approximately 2,000 pesos/USD.

Table 4. OLS and First Differences

	(1)	(2)	(3)	(4)
A. Sales and Expenditures				
	log sales$_{it}$		$\triangle$log sales$_{it}$	
log expenditure$_{it}$	0.675***	0.675***		
	(0.027)	(0.027)		
log labor (ℓ_{it})	0.298***	0.296***		
	(0.039)	(0.040)		
log capital (k_{it})	0.087***	0.087***		
	(0.019)	(0.019)		
$\triangle$ log expenditure$_{it}$			0.488***	0.485***
			(0.050)	(0.052)
$\triangle$ log labor ($\triangle\ell_{it}$)			0.294***	0.288***
			(0.037)	(0.036)
$\triangle$ log capital ($\triangle k_{it}$)			0.010	0.010
			(0.019)	(0.018)
Year effects	N	Y	N	Y
R-squared	0.926	0.927	0.335	0.339
B. Sato-Vartia Output and Input Indexes				
	log output index ($\widetilde{y}_{it}^{SV}$)		$\triangle$ log output index ($\triangle\widetilde{y}_{it}^{SV}$)	
log materials index ($\widetilde{m}_{it}^{SV}$)	0.469***	0.468***		
	(0.084)	(0.085)		
log labor (ℓ_{it})	0.357***	0.358***		
	(0.109)	(0.110)		
log capital (k_{it})	0.196***	0.195***		
	(0.045)	(0.046)		
$\triangle$ log materials index ($\triangle\widetilde{m}_{it}^{SV}$)			0.434***	0.428***
			(0.052)	(0.053)
$\triangle$ log labor ($\triangle\ell_{it}$)			0.283***	0.274***
			(0.046)	(0.045)
$\triangle$ log capital ($\triangle k_{it}$)			0.013	0.015
			(0.020)	(0.020)
Year effects	N	Y	N	Y
R-squared	0.704	0.705	0.258	0.263

Notes: Baseline sample: N (observations) = 4,247, N (distinct firms) = 362 for all regressions. Dependent variables at top of columns. $\triangle$ refers to within-firm difference between $t-1$ and t. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table 5. Differences (Step 1): First Stage

	$\triangle\widetilde{m}_{it}^{SV}$ (1)	$\triangle\ell_{it}$ (2)	$\triangle k_{it}$ (3)	$\triangle\widetilde{m}_{it}^{SV}$ (4)	$\triangle\ell_{it}$ (5)	$\triangle k_{it}$ (6)	$\triangle\widetilde{m}_{it}^{SV}$ (7)	$\triangle\ell_{it}$ (8)	$\triangle k_{it}$ (9)
$\widetilde{m}_{it-2}^{SV}$	-0.017***	0.013***	0.027***				-0.018***	0.013***	0.026***
	(0.006)	(0.004)	(0.005)				(0.006)	(0.004)	(0.005)
ℓ_{it-2}	0.014	-0.030***	0.044***				0.012	-0.031***	0.042***
	(0.009)	(0.007)	(0.010)				(0.009)	(0.007)	(0.010)
k_{it-2}	0.009	0.009**	-0.048***	0.001	0.002	-0.010***	0.007	0.008**	-0.049***
	(0.006)	(0.004)	(0.007)	(0.002)	(0.002)	(0.003)	(0.006)	(0.004)	(0.007)
$\triangle$ pred. import price index ($\triangle\widehat{\widetilde{w}}_{it}^{imp}$)				-0.245**	-0.040	0.073	-0.254***	-0.047	0.119
				(0.097)	(0.064)	(0.103)	(0.098)	(0.063)	(0.102)
$\triangle$ log min. wage x "bite" ($\triangle z_{it}$)				-1.722**	-1.642***	-2.518***	-1.792**	-1.734***	-2.028***
				-0.848	(0.489)	(0.605)	(0.861)	(0.495)	(0.600)
Year effects	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	4,247	4,247	4,247	4,247	4,247	4,247	4,247	4,247	4,247
R-squared	0.024	0.035	0.038	0.025	0.031	0.015	0.026	0.039	0.041
F - statistic	3.424	7.703	21.018	4.486	5.891	6.819	3.704	7.245	13.904
F - SW	2.070	2.366	2.258	4.179	4.988	17.373	5.897	12.195	17.354
KP LM test (underidentification)		1.995			3.925			15.88	
KP Wald F-test (weak insts.)		0.673			1.353			3.363	

Notes: Dependent variables at tops of columns. SW refers to Sanderson and Windmeijer (2016), KP to Kleibergen and Paap (2006). The F-statistic is the standard F for a test that the coefficients on the excluded instruments (indicated at left) are zero. The KP statistics, LM test for under-identification and Wald F test for weak instruments, are for each IV model as a whole, and are not specific to Columns 2, 5, 8. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table 6. Differences (Step 1): Second Stage

	Dep.var.: $\triangle$ log output index ($\triangle\widetilde{y}_{it}^{SV}$)			
	internal instruments only	external instruments + k_{it-2}	internal & external instruments	internal & external instruments (LIML)
	(1)	(2)	(3)	(4)
$\triangle$ log materials index ($\triangle\widetilde{m}_{it}^{SV}$)	0.520	0.247	0.381**	0.381**
	(0.487)	(0.541)	(0.177)	(0.179)
$\triangle$ log labor ($\triangle\ell_{it}$)	0.485	0.600	0.397**	0.397**
	(0.394)	(0.714)	(0.184)	(0.184)
$\triangle$ log capital ($\triangle k_{it}$)	-0.148	-0.211	-0.181	-0.181
	(0.196)	(0.215)	(0.124)	(0.124)
Year effects	Y	Y	Y	Y
N	4,247	4,247	4,247	4,247
R-squared	0.224	0.191	0.239	0.239
Materials Robust (LC) Conf. Int. 90%			[0.150 - 0.612]	[0.103 - 0.658]
Labor Robust (LC) Conf. Int. 90%			[0.159 - 0.636]	[0.105 - 0.690]
Materials Robust CI (LC) Conf. Int. 95%			[0.106 - 0.656]	[0.050 - 0.711]
Labor Robust CI (LC) Conf. Int. 95%			[0.113 - 0.681]	[0.049 - 0.746]
Arellano-Bond AR(2) statistic	0.323	0.285	0.339	0.339
Arellano-Bond p-value	0.746	0.776	0.735	0.735

Notes: Corresponding first-stage estimates are in Table 5: Column 1 here corresponds to Columns 1-3, Column 2 to Columns 4-6, Column 3 to Columns 7-9 of Table 5. Robust standard errors in parentheses. Weak-instrument-robust confidence intervals are based on LC test of Andrews (2018), implemented by Stata twostepweakiv command. Arellano-Bond statistic and p-value test for serial correlation in residual, based on Arellano and Bond (1991). *10% level, **5% level, ***1% level.

Table 7. Levels (Step 2): First and Second Stages

A. First stage

	Dep.var.: log capital (k_{it})	
	(1)	(2)
$\triangle k_{it-1}$	0.666***	
	(0.106)	
$\triangle k_{it-2}$		0.676***
		(0.112)
Year effects	Y	Y
N	4,247	4,131
R squared	0.028	0.027
Kleibergen-Paap LM test (underidentification)	43.269	40.554
Kleibergen-Paap Wald F-test (weak insts.)	39.453	36.727

B. Second stage

	Dep.var.: $\widetilde{y}_{it}^{SV} - \widehat{\beta}_m \widetilde{m}_{it}^{SV} - \widehat{\beta}_\ell \ell_{it}$		
	(1)	(2)	(3)
log capital k_{it}	0.196	0.289	0.239***
	(0.088)	(0.101)	(0.020)
	[0.188]	[0.196]	
Year effects	Y	Y	Y
N	4,247	4,131	4,247
R-squared	0.173	0.170	0.258
Specification	IV	IV	OLS

Notes: Panel B Columns 1-2 correspond to Panel A Columns 1-2. Panel C Column 3 is OLS. Uncorrected robust standard errors in parentheses. Corrected robust standard errors in brackets. See Section 2.4.2 for details. *10% level, **5% level, ***1% level.

Table 8. System GMM

	log sales_{it} (1)	(2)	(3)
log sales_{it-1}	0.636***	0.607***	0.550***
	(0.079)	(0.082)	(0.082)
log expenditure_{it}	0.524***	0.537***	0.483***
	(0.083)	(0.080)	(0.059)
log $\text{expenditure}_{it-1}$	-0.250***	-0.232***	-0.179***
	(0.064)	(0.056)	(0.042)
log labor (ℓ_{it})	0.055	0.051	0.258***
	(0.122)	(0.084)	(0.067)
log labor (ℓ_{it-1})	0.068	0.062	-0.070
	(0.112)	(0.084)	(0.064)
log capital (k_{it})	0.103*	0.050	0.033
	(0.062)	(0.062)	(0.040)
log capital (k_{it-1})	-0.089*	-0.037	-0.014
	(0.050)	(0.053)	(0.034)
N	4,247	4,247	4,247
Lag limit	2	3	all
Hansen test	107.0	156.4	344.2
Hansen p-value	0.428	0.587	1.000

Notes: Table presents estimates of standard System GMM model (Blundell and Bond, 2000), using the "two-step" procedure described in Roodman (2009), with initial weighting matrix defined in Doornik et al. (2012) and finite-sample correction from Windmeijer (2005). The Stata command is xtabond2 (Roodman, 2009), with options h(2), twostep, and robust. The difference equation includes lags to $t-2$ in Column 1, lags to $t-3$ in Column 2, and all available lags in Column 3. The numbers of instruments are as indicated in Appendix Table A12. The Hansen test of overidentifying restrictions is appropriate in the non-homoskedastic case, but should be interpreted with caution, as it is weakened by the presence of many instruments. See Section 6 for further details. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table 9. Proxy-Variable Methods

	Dep.var.: log sales$_{it}$			
	OP	LP	Wooldridge	GNR
	(1)	(2)	(3)	(4)
log expenditure$_{it}$	0.672***	0.636***	0.604***	0.406***
	(0.028)	(0.033)	(0.019)	(0.010)
log labor (ℓ_{it})	0.258***	0.289***	0.290***	0.513***
	(0.049)	(0.046)	(0.010)	(0.035)
log capital (k_{it})	0.131***	0.113***	0.054**	0.138***
	(0.035)	(0.035)	(0.025)	(0.027)
N	1,933	4,247	4,247	4,247

Notes: Table presents estimates in our baseline sample from proxy-variable methods of Olley and Pakes (1996) (OP), Levinsohn and Petrin (2003) (LP), Wooldridge (2009) (using materials as the proxy), and Gandhi et al. (2020) (GNR). OP, LP, and Wooldridge estimates generated by Stata command prodest (Rovigatti and Mollisi, 2018). GNR estimates from authors' own code. See Section 6 for further details. Standard errors in parentheses from bootstraps with 50 replications. *10% level, **5% level, ***1% level.

Table 10. △TFP (Standardized) as Predictor of Future Export Growth

	asinh(export value$_{it+s}$) - asinh(export value$_{it}$)		
	$s = 1$	$s = 2$	$s = 3$
	(1)	(2)	(3)
A. TSIV			
$TFP_t^{TSIV} - TFP_{t-s}^{TSIV}$	0.097	1.204***	1.633***
	(0.310)	(0.321)	(0.365)
R-squared	0.005	0.012	0.017
B. OLS using Sato-Vartia quantity indexes			
$TFP_t^{OLS-SV} - TFP_{t-s}^{OLS-SV}$	0.074	0.437***	0.463**
	(0.152)	(0.156)	(0.181)
R-squared	0.005	0.010	0.011
C. System GMM			
$TFP_t^{SysGMM} - TFP_{t-s}^{SysGMM}$	0.158	0.753***	0.971***
	(0.210)	(0.219)	(0.254)
R-squared	0.005	0.012	0.015
D. Olley and Pakes			
$TFP_t^{OP} - TFP_{t-s}^{OP}$	0.071	0.329**	0.411**
	(0.138)	(0.145)	(0.170)
R-squared	0.005	0.009	0.011
E. Levinsohn and Petrin			
$TFP_t^{LP} - TFP_{t-s}^{LP}$	0.071	0.350**	0.427**
	(0.139)	(0.146)	(0.171)
R-squared	0.005	0.009	0.011
F. Wooldridge			
$TFP_t^{W} - TFP_{t-s}^{W}$	0.091	0.452***	0.577***
	(0.154)	(0.162)	(0.190)
R-squared	0.005	0.010	0.012
G. Gandhi, Navarro and Rivers			
$TFP_t^{GNR} - TFP_{t-s}^{GNR}$	0.035	0.460***	0.463**
	(0.155)	(0.159)	(0.185)
R-squared	0.005	0.010	0.011
H. Ackerberg, Caves and Frazer			
$TFP_t^{ACF} - TFP_{t-s}^{ACF}$	0.149	0.343**	0.353**
	(0.126)	(0.135)	(0.152)
R-squared	0.005	0.010	0.010
N	3,539	2,835	2,127
Year effects	Y	Y	Y

Notes: TFP calculated in levels (see Appendix Table A14 for details and pairwise correlations), then standardized by year to have variance 1 (pooling years). (For OP method, TFP calculated for all observations in baseline sample, even those with zero investment omitted from estimation in Table 9.) Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Estimating Production Functions in Differentiated-Product Industries with Quantity Information and External Instruments*

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Dec. 2020

Online Appendix

A Theory Appendix

A.1 Construction of CES Price/Quantity Indexes, Output Side

A.1.1 Consumer's Minimization Problem

The Lagrangian corresponding to the first stage of the consumer's problem is given by:

$$\mathcal{L}^y = \sum_{j\in\Omega_{it}^y} Y_{ijt}P_{ijt} - \lambda\left(\left[\sum_{j\in\Omega_{it}^y} (\varphi_{ijt}Y_{ijt})^{\frac{\sigma_i^y-1}{\sigma_i^y}}\right]^{\frac{\sigma_i^y}{\sigma_i^y-1}} - \widetilde{Y}_{it}\right)$$

The first order condition with respect to product j, $\frac{\partial \mathcal{L}^y}{\partial Y_{ijt}} = 0$, implies:

$$\frac{P_{ijt}}{\varphi_{ijt}} = \lambda(\varphi_{ijt}Y_{ijt})^{-\frac{1}{\sigma_i^y}}\widetilde{Y}_{it}^{\frac{1}{\sigma_i^y}} \tag{A1}$$

Raising both sides of this equation to the power $1-\sigma_i^y$, summing over the $j\in\Omega_{it}^y$, using the definition of $\widetilde{P}_{it}$ in (2), and rearranging, we have:

$$\lambda = \widetilde{P}_{it} \tag{A2}$$

The second-order conditions for minimization are satisfied without further assumptions if and only if $\sigma_i^y \in (0,1)\cup(1,\infty)$.[1] To see this, note that a necessary and sufficient condition for $\mathcal{L}^y$ to be convex is that all principal minors of order r of the Hessian matrix of $\mathcal{L}^y$ are non-negative, for $r = 1,\cdots,J$, where J is the number of products in Ω_{it}^y. (See e.g. Theorem 2.3.3 in Sydsaeter et al. (2005).) Chen (2012) shows (Theorem 5.1) that the determinant of the Hessian matrix of a CES function is always zero. This implies that the principal minor of order J of the Hessian for $\mathcal{L}^y$ is zero. Furthermore, every principal minor of degree $1 \le r < J-1$ corresponds to the determinant of the Hessian matrix of a CES aggregator with $J-r$ varieties and hence is also zero.[2] We are left only with the principal minors of order one, which correspond to the elements of the diagonal of the Hessian matrix of $\mathcal{L}^y$, which are the second derivatives:

$$\frac{\partial^2\mathcal{L}^y}{\partial^2 Y_{ijt}} = \frac{-\lambda}{\sigma_i^y}\left[\widetilde{Y}_{it}^{\frac{1}{\sigma_i^y}}\varphi_{ijt}^{\frac{\sigma_i^y-1}{\sigma_i^y}}Y_{ijt}^{\frac{-1-\sigma_i^y}{\sigma_i^y}}\right]\left[\left(\frac{\varphi_{ijt}Y_{ijt}}{\widetilde{Y}_{it}}\right)^{\frac{\sigma_i^y-1}{\sigma_i^y}} - 1\right]$$

Given that the second term in brackets is always negative and $\lambda > 0$ (see (A2)), all principal minors of order one are greater than zero if and only if $\sigma_i^y \in (0,1)\cup(1,\infty)$. Hence $\mathcal{L}^y$ is convex and the second-order conditions for minimization are satisfied for a critical point satisfying the Lagrangian first order conditions if and only if $\sigma_i^y \in (0,1)\cup(1,\infty)$. In the terminology of Sun and Yang (2006), goods in the bundle Ω_{it}^y are *gross substitutes* when $\sigma_i^y > 1$ and *gross complements* when $0 < \sigma_i^y < 1$. That is, the demand for product j increases in response to an increase in the price of any other variety k, holding everything else constant, if and only if $\sigma_i^y > 1$; it decreases

[1] For the limiting case $\sigma_i^y \to 1$, $\widetilde{Y}_{it}$ tends to a Cobb-Douglas function with exponents φ_{ijt}. In this case, $\widetilde{Y}_{it}$ is concave if and only if $\sum_{j\in\Omega_{it}^y}\varphi_{ijt} \le 1$.

[2] Note that the theorem still applies when replacing $p(x)$ by $p(x)+c$, where the additional constant arises due to the excluded varieties that now enter as constant terms within the sum.

if and only if $0 < \sigma_i^y < 1$. Although our methodology can accommodate either case, we believe that given the sectors we consider in our empirical application, it is reasonable to assume $\sigma_i^y > 1$. In other settings, it may be plausible to allow $0 < \sigma_i^y < 1$.

Plugging (A2) into (A1) and rearranging, we can express the output quantity for product j in terms of its price, its quality, and the firm-level aggregate output and price index:

$$Y_{ijt} = \widetilde{Y}_{it} \left(\frac{\widetilde{P}_{it}}{P_{ijt}} \right)^{\sigma_i^y} {\varphi_{ijt}}^{\sigma_i^y - 1} \tag{A3}$$

Note that:

$$R_{it} = \sum_{j \in \Omega_{it}^y} R_{ijt} = \sum_{j \in \Omega_{it}^y} P_{ijt} Y_{ijt} = \widetilde{P}_{it} \widetilde{Y}_{it} \left(\widetilde{P}_{it} \right)^{\sigma_i^y - 1} \underbrace{\sum_{j \in \Omega_{it}^y} \left(\frac{P_{ijt}}{\varphi_{ijt}} \right)^{1-\sigma_i^y}}_{=\widetilde{P}_{it}^{1-\sigma_i^y}} = \widetilde{P}_{it} \widetilde{Y}_{it} \tag{A4}$$

That is, $\widetilde{P}_{it}$ is indeed the price index that sets $R_{it} = \widetilde{P}_{it} \widetilde{Y}_{it}$.

A.1.2 Price Index Log Change

Using (A3),

$$S_{ijt}^y = \frac{P_{ijt} Y_{ijt}}{R_{it}} = \frac{P_{ijt} Y_{ijt}}{\widetilde{P}_{it} \widetilde{Y}_{it}} = \left(\frac{\left(\frac{P_{ijt}}{\varphi_{ijt}} \right)}{\widetilde{P}_{it}} \right)^{1-\sigma_i^y} \tag{A5}$$

Hence from the definitions in (5) in the main text:

$$\chi_{it,t-1}^y = \frac{\sum_{j \in \Omega_{it}^{y*}} S_{ijt}^y}{\sum_{j \in \Omega_{it}^y} S_{ijt}^y} = \frac{\sum_{j \in \Omega_{it}^{y*}} \left(\frac{P_{ijt}}{\varphi_{ijt}} \right)^{1-\sigma_i^y}}{\sum_{j \in \Omega_{it}^y} \left(\frac{P_{ijt}}{\varphi_{ijt}} \right)^{1-\sigma_i^y}}, \quad \chi_{it-1,t}^y = \frac{\sum_{j \in \Omega_{it}^{y*}} S_{ijt-1}^y}{\sum_{j \in \Omega_{it-1}^y} S_{ijt-1}^y} = \frac{\sum_{j \in \Omega_{it}^{y*}} \left(\frac{P_{ijt-1}}{\varphi_{ijt-1}} \right)^{1-\sigma_i^y}}{\sum_{j \in \Omega_{it-1}^y} \left(\frac{P_{ijt-1}}{\varphi_{ijt-1}} \right)^{1-\sigma_i^y}}$$

Then using the definition of $\widetilde{P}_{it}$, (2),

$$\begin{aligned} \frac{\widetilde{P}_{it}}{\widetilde{P}_{it-1}} &= \frac{\left[\sum_{j \in \Omega_{it}^y} \left(\frac{P_{ijt}}{\varphi_{ijt}} \right)^{1-\sigma_i^y} \right]^{\frac{1}{1-\sigma_i^y}}}{\left[\sum_{j \in \Omega_{it-1}^y} \left(\frac{P_{ijt-1}}{\varphi_{ijt-1}} \right)^{1-\sigma_i^y} \right]^{\frac{1}{1-\sigma_i^y}}} \\ &= \left(\frac{\chi_{it-1,t}^y}{\chi_{it,t-1}^y} \right)^{\frac{1}{1-\sigma_i^y}} \frac{\left(\sum_{j \in \Omega_{it}^{y*}} \left(\frac{P_{ijt}}{\varphi_{ijt}} \right)^{1-\sigma_i^y} \right)^{\frac{1}{1-\sigma_i^y}}}{\left(\sum_{j \in \Omega_{it}^{y*}} \left(\frac{P_{ijt-1}}{\varphi_{ijt-1}} \right)^{1-\sigma_i^y} \right)^{\frac{1}{1-\sigma_i^y}}} = \left(\frac{\chi_{it-1,t}^y}{\chi_{it,t-1}^y} \right)^{\frac{1}{1-\sigma_i^y}} \frac{\widetilde{P}_{it}^*}{\widetilde{P}_{it-1}^*} \end{aligned} \tag{A6}$$

where $\widetilde{P}_{it}^*$ is the common-goods price index defined in the main text (footnote 15).

To derive an expression for $\frac{\widetilde{P}_{it}^*}{\widetilde{P}_{it-1}^*}$, note that (A5) implies a similar expression for the expenditure share of common goods:

$$S_{ijt}^{y*} = \frac{P_{ijt}Y_{ijt}}{\widetilde{P}_{it}^{*}\widetilde{Y}_{it}^{*}} = \frac{P_{ijt}Y_{ijt}}{\widetilde{P}_{it}\widetilde{Y}_{it}} \cdot \frac{\widetilde{P}_{it}\widetilde{Y}_{it}}{\widetilde{P}_{it}^{*}\widetilde{Y}_{it}^{*}} = \left(\frac{\left(\frac{P_{ijt}}{\varphi_{ijt}}\right)}{\widetilde{P}_{it}}\right)^{1-\sigma_i^y} \frac{\widetilde{P}_{it}\widetilde{Y}_{it}}{\widetilde{P}_{it}^{*}\widetilde{Y}_{it}^{*}}$$

Using (A3),

$$\frac{\widetilde{P}_{it}\widetilde{Y}_{it}}{\widetilde{P}_{it}^{*}\widetilde{Y}_{it}^{*}} = \frac{\widetilde{P}_{it}\widetilde{Y}_{it}}{\sum_{j\in\Omega_{it}^{y*}} P_{ijt}Y_{ijt}} = \left(\frac{\widetilde{P}_{it}}{\widetilde{P}_{it}^{*}}\right)^{1-\sigma_i^y}$$

Hence:

$$S_{ijt}^{y*} = \left(\frac{\left(\frac{P_{ijt}}{\varphi_{ijt}}\right)}{\widetilde{P}_{it}^{*}}\right)^{1-\sigma_i^y} \tag{A7}$$

Divide (A7) by the same equation for the previous year, take logs, and re-arrange:

$$\frac{\ln\left(\frac{\widetilde{P}_{it}^{*}}{\widetilde{P}_{it-1}^{*}}\right) - \ln\left(\frac{\frac{P_{ijt}}{\varphi_{ijt}}}{\frac{P_{ijt-1}}{\varphi_{ijt-1}}}\right)}{\ln\left(\frac{S_{ijt}^{y*}}{S_{ijt-1}^{y*}}\right)} = \frac{1}{\sigma_i^y - 1}$$

Multiply both sides by $S_{ijt}^{y*} - S_{ijt-1}^{y*}$ and sum over the common goods:

$$\sum_{j\in\Omega_{it}^{y*}} \left(S_{ijt}^{y*} - S_{ijt-1}^{y*}\right) \frac{\ln\left(\frac{\widetilde{P}_{it}^{*}}{\widetilde{P}_{it-1}^{*}}\right) - \ln\left(\frac{\frac{P_{ijt}}{\varphi_{ijt}}}{\frac{P_{ijt-1}}{\varphi_{ijt-1}}}\right)}{\ln\left(\frac{S_{ijt}^{y*}}{S_{ijt-1}^{y*}}\right)} = \left(\frac{1}{\sigma_i^y - 1}\right) \sum_{j\in\Omega_{it}^{y*}} \left(S_{ijt}^{y*} - S_{ijt-1}^{y*}\right) = 0$$

where the second equality follows because $\sum_{j\Omega_{it}^{y*}} S_{ijt}^{y*} = \sum_{j\Omega_{it}^{y*}} S_{ijt-1}^{y*} = 1$. This implies:

$$\sum_{j\in\Omega_{it}^{y*}} \left(\frac{S_{ijt}^{y*} - S_{ijt-1}^{y*}}{\ln S_{ijt}^{y*} - \ln S_{ijt-1}^{y*}}\right) \ln\left(\frac{\widetilde{P}_{it}^{*}}{\widetilde{P}_{it-1}^{*}}\right) = \sum_{j\in\Omega_{it}^{y*}} \left(\frac{S_{ijt}^{y*} - S_{ijt-1}^{y*}}{\ln S_{ijt}^{y*} - \ln S_{ijt-1}^{y*}}\right) \ln\left(\frac{\frac{P_{ijt}}{\varphi_{ijt}}}{\frac{P_{ijt-1}}{\varphi_{ijt-1}}}\right).$$

Since $\ln\left(\frac{\widetilde{P}_{it}^{*}}{\widetilde{P}_{it-1}^{*}}\right)$ does not vary with j, this can be re-written as:

$$\ln\left(\frac{\widetilde{P}_{it}^{*}}{\widetilde{P}_{it-1}^{*}}\right) = \sum_{j\in\Omega_{it}^{y*}} \delta_{ijt} \ln\left(\frac{P_{ijt}}{P_{ijt-1}}\right) - \sum_{j\in\Omega_{it}^{y*}} \delta_{ijt} \ln\left(\frac{\varphi_{ijt}}{\varphi_{ijt-1}}\right), \tag{A8}$$

where δ_{ijt} is as defined in (5) above. Combining (A6) and (A8), we have:

$$\ln\left(\frac{\widetilde{P}_{it}}{\widetilde{P}_{it-1}}\right) = \sum_{j\in\Omega_{it}^{y*}} \delta_{ijt} \ln\left(\frac{P_{ijt}}{P_{ijt-1}}\right) - \sum_{j\in\Omega_{it}^{y*}} \delta_{ijt} \ln\left(\frac{\varphi_{ijt}}{\varphi_{ijt-1}}\right) - \frac{1}{\sigma_i^y - 1} \ln\left(\frac{\chi_{it-1,t}^y}{\chi_{it,t-1}^y}\right) \tag{A9}$$

which is (4).

A.1.3 Quantity Index Log Change

To derive the log change in the quantity index, start by noting that (A3) implies,

$$P_{ijt} = \widetilde{P}_{it} \left(\frac{\widetilde{Y}_{it}}{Y_{ijt}}\right)^{\frac{1}{\sigma_i^y}} \varphi_{ijt}^{\frac{\sigma_i^y - 1}{\sigma_i^y}}$$

Therefore,

$$\ln\left(\frac{P_{ijt}}{P_{ijt-1}}\right) = \ln\left(\frac{\widetilde{P}_{it}}{\widetilde{P}_{it-1}}\right) + \frac{1}{\sigma_i^y} \ln\left(\frac{\widetilde{Y}_{it}}{\widetilde{Y}_{it-1}}\right) - \frac{1}{\sigma_i^y} \ln\left(\frac{Y_{ijt}}{Y_{ijt-1}}\right) + \frac{\sigma_i^y}{\sigma_i^y - 1} \ln\left(\frac{\varphi_{ijt}}{\varphi_{ijt-1}}\right)$$

Plugging this into (A9), re-arranging, and using the fact that $\sum_{j\in\Omega_{it}^{y*}} \delta_{ijt} = 1$ gives:

$$\ln\left(\frac{\widetilde{Y}_{it}}{\widetilde{Y}_{it-1}}\right) = \sum_{j\in\Omega_{it}^{y*}} \delta_{ijt} \ln\left(\frac{Y_{ijt}}{Y_{ijt-1}}\right) + \sum_{j\in\Omega_{it}^{y*}} \delta_{ijt} \ln\left(\frac{\varphi_{ijt}}{\varphi_{ijt-1}}\right) + \frac{\sigma_i^y}{\sigma_i^y - 1} \ln\left(\frac{\chi_{it-1,t}^y}{\chi_{it,t-1}^y}\right)$$

which is (6). The fact that $\widetilde{P}_{it}^* \widetilde{Y}_{it}^* = R_{it}^*$ can be shown as in (A4), using just common goods.

A.2 Construction of CES Price/Quantity Indexes, Input Side

The derivations for the price and quantity indexes for the input side are analogous to the ones from the output side. We include them for the sake of completeness.

A.2.1 Firm's Minimization Problem

The Lagrangian corresponding to the first stage of the firm's problem is given by:

$$\mathcal{L}^m = \sum_{h\in\Omega_{it}^m} M_{iht} W_{iht} - \lambda \left(\left[\sum_{h\in\Omega_{it}^m} (\alpha_{iht} M_{iht})^{\frac{\sigma_i^m - 1}{\sigma_i^m}} \right]^{\frac{\sigma_i^m}{\sigma_i^m - 1}} - \widetilde{M}_{it} \right)$$

The first order condition with respect to input h, $\frac{\partial \mathcal{L}^m}{\partial M_{iht}} = 0$, implies:

$$\frac{W_{iht}}{\alpha_{iht}} = \lambda (\alpha_{iht} M_{iht})^{-\frac{1}{\sigma_i^m}} \widetilde{M}_{it}^{\frac{1}{\sigma_i^m}} \tag{A10}$$

Raising both sides of this equation to the power $1 - \sigma_i^m$, summing over the $h \in \Omega_{it}^m$, using the definition of $\widetilde{W}_{it}$ in (9) in the main text, and rearranging, we have:

$$\lambda = \widetilde{W}_{it} \tag{A11}$$

Analogously to the output case, it can be shown that (without further assumptions) any point satisfying the first order conditions constitutes an global minimum if and only if $\sigma_i^m \in (0,1) \cup (1,\infty)$. Therefore, our method allows material inputs to be *gross complements*, $\sigma_i^m < 1$, or to be *gross substitutes*, $\sigma_i^m > 1$. Nevertheless, given the type of sectors we consider in our empirical analysis, we assume material inputs to be *gross substitutes*, that is, we assume $\sigma_i^m > 1$.

Plugging (A11) into (A10) and rearranging:

$$M_{iht} = \widetilde{M}_{it} \left(\frac{\widetilde{W}_{it}}{W_{iht}} \right)^{\sigma_i^m} {\alpha_{iht}}^{\sigma_i^m - 1} \tag{A12}$$

As for revenues,

$$E_{it} = \sum_{h \in \Omega_{it}^m} E_{iht} = \sum_{h \in \Omega_{it}^m} W_{iht} M_{iht} = \widetilde{W}_{it} \widetilde{M}_{it} \left(\widetilde{W}_{it} \right)^{\sigma_i^m - 1} \underbrace{\sum_{h \in \Omega_{it}^m} \left(\frac{W_{iht}}{\alpha_{iht}} \right)^{1-\sigma_i^m}}_{= \widetilde{W}_{it}^{1-\sigma_i^m}} = \widetilde{W}_{it} \widetilde{M}_{it} \tag{A13}$$

A.2.2 Price Index Log Change

Using (A12),

$$S_{iht}^m = \frac{W_{iht} M_{iht}}{E_{it}} = \frac{W_{iht} M_{iht}}{\widetilde{W}_{it} \widetilde{M}_{it}} = \left(\frac{\left(\frac{W_{iht}}{\alpha_{iht}} \right)}{\widetilde{W}_{it}} \right)^{1-\sigma_i^m} \tag{A14}$$

Hence from the definitions in (11) in the main text:

$$\chi_{it,t-1}^m = \frac{\sum_{h \in \Omega_{it}^{m*}} S_{iht}^m}{\sum_{h \in \Omega_{it}^m} S_{iht}^m} = \frac{\sum_{h \in \Omega_{it}^{m*}} \left(\frac{W_{iht}}{\alpha_{iht}} \right)^{1-\sigma_i^m}}{\sum_{h \in \Omega_{it}^m} \left(\frac{W_{iht}}{\alpha_{iht}} \right)^{1-\sigma_i^m}}, \; \chi_{it-1,t}^m = \frac{\sum_{h \in \Omega_{it}^{m*}} S_{iht-1}^m}{\sum_{h \in \Omega_{it-1}^m} S_{iht-1}^m} = \frac{\sum_{h \in \Omega_{it}^{m*}} \left(\frac{W_{iht-1}}{\alpha_{iht-1}} \right)^{1-\sigma_i^m}}{\sum_{h \in \Omega_{it-1}^m} \left(\frac{W_{iht-1}}{\alpha_{iht-1}} \right)^{1-\sigma_i^m}}$$

Then using the definition of $\widetilde{W}_{it}$, (9),

$$\begin{aligned} \frac{\widetilde{W}_{it}}{\widetilde{W}_{it-1}} &= \frac{\left[\sum_{h \in \Omega_{it}^m} \left(\frac{W_{iht}}{\alpha_{iht}} \right)^{1-\sigma_i^m} \right]^{\frac{1}{1-\sigma_i^m}}}{\left[\sum_{h \in \Omega_{it-1}^m} \left(\frac{W_{iht-1}}{\alpha_{iht-1}} \right)^{1-\sigma_i^m} \right]^{\frac{1}{1-\sigma_i^m}}} \\ &= \left(\frac{\chi_{it-1,t}^m}{\chi_{it,t-1}^m} \right)^{\frac{1}{1-\sigma_i^m}} \frac{\left(\sum_{h \in \Omega_{it}^{m*}} \left(\frac{W_{iht}}{\alpha_{iht}} \right)^{1-\sigma_i^m} \right)^{\frac{1}{1-\sigma_i^m}}}{\left(\sum_{h \in \Omega_{it}^{m*}} \left(\frac{W_{iht-1}}{\alpha_{iht-1}} \right)^{1-\sigma_i^m} \right)^{\frac{1}{1-\sigma_i^m}}} = \left(\frac{\chi_{it-1,t}^m}{\chi_{it,t-1}^m} \right)^{\frac{1}{1-\sigma_i^m}} \frac{\widetilde{W}_{it}^*}{\widetilde{W}_{it-1}^*} \end{aligned} \tag{A15}$$

where $\widetilde{W}_{it}^{*}$ is the common-goods price index defined in the main text (footnote 22).

To derive an expression for $\frac{\widetilde{W}_{it}^{*}}{\widetilde{W}_{it-1}^{*}}$, note that (A14) implies a similar expression for the expenditure share of common goods:

$$S_{iht}^{m*} = \frac{W_{iht}M_{iht}}{\widetilde{W}_{it}^{*}\widetilde{M}_{it}^{*}} = \frac{W_{iht}M_{iht}}{\widetilde{W}_{it}\widetilde{M}_{it}} \cdot \frac{\widetilde{W}_{it}\widetilde{M}_{it}}{\widetilde{W}_{it}^{*}\widetilde{M}_{it}^{*}} = \left(\frac{\left(\frac{W_{iht}}{\alpha_{iht}}\right)}{\widetilde{W}_{it}}\right)^{1-\sigma_i^m} \frac{\widetilde{W}_{it}\widetilde{M}_{it}}{\widetilde{W}_{it}^{*}\widetilde{M}_{it}^{*}}$$

Using (A3),

$$\frac{\widetilde{W}_{it}\widetilde{M}_{it}}{\widetilde{W}_{it}^{*}\widetilde{M}_{it}^{*}} = \frac{\widetilde{W}_{it}\widetilde{M}_{it}}{\sum_{h\in\Omega_{it}^{m*}} W_{iht}M_{iht}} = \left(\frac{\widetilde{W}_{it}}{\widetilde{W}_{it}^{*}}\right)^{1-\sigma_i^m}$$

Hence:

$$S_{iht}^{m*} = \left(\frac{\left(\frac{W_{iht}}{\alpha_{iht}}\right)}{\widetilde{W}_{it}^{*}}\right)^{1-\sigma_i^m} \tag{A16}$$

Divide (A16) by the same equation for the previous year, take logs, and re-arrange:

$$\frac{\ln\left(\frac{\widetilde{W}_{it}^{*}}{\widetilde{W}_{it-1}^{*}}\right) - \ln\left(\frac{\frac{W_{iht}}{\alpha_{iht}}}{\frac{W_{iht-1}}{\alpha_{iht-1}}}\right)}{\ln\left(\frac{S_{iht}^{m*}}{S_{iht-1}^{m*}}\right)} = \frac{1}{\sigma_i^m - 1}$$

Multiply both sides by $S_{iht}^{m*} - S_{iht-1}^{m*}$ and sum over the common goods:

$$\sum_{h\in\Omega_{it}^{m*}} \left(S_{iht}^{m*} - S_{iht-1}^{m*}\right) \frac{\ln\left(\frac{\widetilde{W}_{it}^{*}}{\widetilde{W}_{it-1}^{*}}\right) - \ln\left(\frac{\frac{W_{iht}}{\alpha_{iht}}}{\frac{W_{iht-1}}{\alpha_{iht-1}}}\right)}{\ln\left(\frac{S_{iht}^{m*}}{S_{iht-1}^{m*}}\right)} = \left(\frac{1}{\sigma_i^m - 1}\right) \sum_{h\in\Omega_{it}^{m*}} \left(S_{iht}^{m*} - S_{iht-1}^{m*}\right) = 0$$

where the second equality follows because $\sum_{h\in\Omega_{it}^{m*}} S_{iht}^{m*} = \sum_{h\in\Omega_{it}^{m*}} S_{iht-1}^{m*} = 1$. This implies:

$$\sum_{h\in\Omega_{it}^{m*}} \left(\frac{S_{iht}^{m*} - S_{iht-1}^{m*}}{\ln S_{iht}^{m*} - \ln S_{iht-1}^{m*}}\right) \ln\left(\frac{\widetilde{W}_{it}^{*}}{\widetilde{W}_{it-1}^{*}}\right) = \sum_{h\in\Omega_{it}^{m*}} \left(\frac{S_{iht}^{m*} - S_{iht-1}^{m*}}{\ln S_{iht}^{m*} - \ln S_{iht-1}^{m*}}\right) \ln\left(\frac{\frac{W_{iht}}{\alpha_{iht}}}{\frac{W_{iht-1}}{\alpha_{iht-1}}}\right)$$

Since $\ln\left(\frac{\widetilde{W}_{it}^{*}}{\widetilde{W}_{it-1}^{*}}\right)$ does not vary with h, this can be re-written as:

$$\ln\left(\frac{\widetilde{W}_{it}^{*}}{\widetilde{W}_{it-1}^{*}}\right) = \sum_{h\in\Omega_{it}^{m*}} \delta_{iht} \ln\left(\frac{W_{iht}}{W_{iht-1}}\right) - \sum_{h\in\Omega_{it}^{m*}} \delta_{iht} \ln\left(\frac{\alpha_{iht}}{\alpha_{iht-1}}\right) \tag{A17}$$

where δ_{iht} is as defined in (11) above. Combining (A15) and (A17), we have:

$$\ln\left(\frac{\widetilde{W}_{it}}{\widetilde{W}_{it-1}}\right) = \sum_{h\in\Omega_{it}^{m*}} \delta_{iht} \ln\left(\frac{W_{iht}}{W_{iht-1}}\right) - \sum_{h\in\Omega_{it}^{m*}} \delta_{iht} \ln\left(\frac{\alpha_{iht}}{\alpha_{iht-1}}\right) - \frac{1}{\sigma_i^m - 1} \ln\left(\frac{\chi_{it-1,t}^m}{\chi_{it,t-1}^m}\right) \quad \text{(A18)}$$

which is (10) in the main text.

A.2.3 Quantity Index Log Change

We start by noting that (A12) implies

$$W_{iht} = \widetilde{W}_{it} \left(\frac{\widetilde{M}_{it}}{M_{iht}}\right)^{\frac{1}{\sigma_i^m}} \alpha_{iht}^{\frac{\sigma_i^m - 1}{\sigma_i^m}}$$

Hence:

$$\ln\left(\frac{W_{iht}}{W_{iht-1}}\right) = \ln\left(\frac{\widetilde{W}_{it}}{\widetilde{W}_{it-1}}\right) + \frac{1}{\sigma_i^y} \ln\left(\frac{\widetilde{M}_{it}}{\widetilde{M}_{it-1}}\right) - \frac{1}{\sigma_i^y} \ln\left(\frac{W_{iht}}{W_{iht-1}}\right) + \frac{\sigma_i^y}{\sigma_i^y - 1} \ln\left(\frac{\alpha_{iht}}{\alpha_{iht-1}}\right)$$

Plugging this into (A18), re-arranging, and using the fact that $\sum_{h\in\Omega_{it}^{m*}} \delta_{iht} = 1$ gives the log change in $\widetilde{M}_{it}$:

$$\ln\left(\frac{\widetilde{M}_{it}}{\widetilde{M}_{it-1}}\right) = \sum_{h\in\Omega_{it}^{m*}} \delta_{iht} \ln\left(\frac{M_{iht}}{M_{iht-1}}\right) + \sum_{h\in\Omega_{it}^{m*}} \delta_{iht} \ln \frac{\alpha_{iht}}{\alpha_{iht-1}} + \frac{\sigma_i^m}{\sigma_i^m - 1} \ln\left(\frac{\chi_{it-1,t}^m}{\chi_{it,t-1}^m}\right)$$

which is (12) in the main text. The fact that $\widetilde{W}_{it}^* \widetilde{M}_{it}^* = E_{it}^*$ can be shown as in (A13), using just common goods.

A.3 Variance Correction for β_k in Levels-Equation Estimation

Our sequential production function estimation belongs to a general class of two-step M-Estimators discussed for instance in Wooldridge (2002, Section 12.4) (and previously in Newey (1984)). The results there can be applied directly. Under our assumptions, our first-step estimates $\widehat{\beta}_m$ and $\widehat{\beta}_l$ and their standard errors are consistently estimated. The levels-equation estimate of β_k, call it $\widehat{\widehat{\beta}}_k$, can be calculated by solving:

$$\sum_{t=1}^{T} \sum_{i=1}^{N} \triangle k_{it-1} \left(\left(\widetilde{y}_{it}^{SV} - \widehat{\beta}_m \widetilde{m}_{it}^{SV} - \widehat{\beta}_l l_{it}\right) - \widehat{\widehat{\beta}}_k k_{it} \right) = 0. \quad \text{(A19)}$$

As noted in the main text (see footnote 38), the consistency of $\widehat{\beta}_m$ and $\widehat{\beta}_l$ is sufficient to guarantee the consistency of $\widehat{\widehat{\beta}}_k$. In the special case when $E(\triangle k_{it-1}\widetilde{m}_{it}^{SV}) = 0$ and $E(\triangle k_{it-1}\ell_{it}) = 0$, the first step estimation can be ignored when computing the asymptotic variance of $\widehat{\widehat{\beta}}_k$.[3] If those

[3]The score function corresponding to the levels-equation IV estimation is $s(a_{it}, \beta_k; \beta_m, \beta_l) = \triangle k_{it-1} (\widetilde{y}_{it}^{SV} - \beta_m \widetilde{m}_{it}^{SV} - \beta_l l_{it} - \beta_k k_{it})$, where $a_{it} = (\widetilde{y}_{it}^{SV}, \widetilde{m}_{it}^{SV}, l_{it}, k_{it}, \triangle k_{it-1})$. If $E(\triangle k_{it-1}\widetilde{m}_{it}^{SV}) = 0$ and $E(\triangle k_{it-1}\ell_{it}) = 0$ then the gradient of the score function with respect to β_m and β_ℓ is zero and equation 12.37 of Wooldridge (2002) holds,

conditions do not hold, then we need to use a corrected expression for the asymptotic variance of $\widehat{\beta}_k$, which takes into account that $\widehat{\beta}_m$ and $\widehat{\beta}_l$ were estimated in a previous step. A consistent estimate of the corrected asymptotic variance for $\widehat{\widehat{\beta}}_k$, call it $\widehat{V}_{\beta_k}$, is given by Newey and McFadden (1994):[4]

$$\widehat{V}_{\beta_k} = \frac{(T \times N)^{-1} \left(\sum_{t=1}^{T} \sum_{i=1}^{N} \left(\widehat{s}_{it} + \widehat{F} \widehat{\psi}_{it} \right)^2 \right)}{\widehat{G}^2} \tag{A20}$$

where

$$\begin{aligned}
\widehat{G} &= -\frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \triangle k_{it-1} k_{it} \\
\widehat{F} &= -\frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \triangle k_{it-1} \left[\widetilde{m}_{it}^{SV}, l_{it}, 0 \right] \\
\widehat{s}_{it} &= \triangle k_{it-1} \left(\widetilde{y}_{it}^{SV} - \widehat{\beta}_m \widetilde{m}_{it}^{SV} - \widehat{\beta}_l l_{it} - \widehat{\widehat{\beta}}_k k_{it} \right) \\
\widehat{\psi}_{it} &= -\left(\widehat{H}' \widehat{W} \widehat{H} \right)^{-1} \widehat{H}' \widehat{W} \widehat{m}_{it}
\end{aligned}$$

and the terms in $\widehat{\psi}_{it}$ are defined as:

$$\begin{aligned}
\widehat{H} &= \frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \left[\triangle \widehat{\widetilde{w}}_{it}^{imp}, \triangle z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right] \left[\triangle \widetilde{m}_{it}^{SV}, \triangle l_{it}, \triangle k_{it} \right]' \\
\widehat{W} &= \frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \left[\triangle \widehat{\widetilde{w}}_{it}^{imp}, \triangle z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right] \left[\triangle \widehat{\widetilde{g}}_{it}, \triangle z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right]' \\
\widehat{m}_{it} &= \left[\triangle \widehat{\widetilde{g}}_{it}, \triangle z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right]' \left(\triangle \widetilde{y}_{it}^{SV} - \widehat{\beta}_m \triangle \widetilde{m}_{it}^{SV} - \widehat{\beta}_l \triangle l_{it} - \widehat{\beta}_k \triangle k_{it} \right)
\end{aligned}$$

We report the corresponding corrected standard errors when we report $\widehat{\widehat{\beta}}_k$.

A.4 Construction of Alternative Quantity Indexes

On the input side, following standard formulations (see e.g. Dodge (2008)), we define the Laspeyres input quantity index for $t-1$ and t as:

$$\widetilde{M}_{it,t-1}^{Lasp} = \frac{\sum_{h \in \Omega_{it}^{m*}} W_{iht-1} M_{iht}}{\sum_{h \in \Omega_{it}^{m*}} W_{iht-1} M_{iht-1}} \tag{A21}$$

and the Paasche input quantity index as:

$$\widetilde{M}_{it,t-1}^{Paas} = \frac{\sum_{h \in \Omega_{it}^{m*}} W_{iht} M_{iht}}{\sum_{h \in \Omega_{it}^{m*}} W_{iht} M_{iht-1}}, \tag{A22}$$

implying that we can ignore the first step in calculating the asymptotic variance of $\widehat{\widehat{\beta}}_k$.

[4] See also Proposition 2 of Kripfganz and Schwarz (2019).

The Tornqvist quantity index is defined as:

$$\widetilde{M}^{Torn}_{it,t-1} = \prod_{h \in \Omega^{m*}_{it}} \left(\frac{M_{iht}}{M_{iht-1}} \right)^{\frac{1}{2}(S^{m*}_{iht} + S^{m*}_{iht-1})} \tag{A23}$$

where S^{m*}_{iht} and S^{m*}_{iht-1} are as defined in footnote 22 of the main text.

Note that the Laspeyres quantity index is related to the Paasche price index, and vice-versa. If we define the Laspeyres price index as:

$$\widetilde{W}^{Lasp}_{it,t-1} = \frac{\sum_{h \in \Omega^{m*}_{it}} W_{iht} M_{iht-1}}{\sum_{h \in \Omega^{m*}_{it}} W_{iht-1} M_{iht-1}}. \tag{A24}$$

and the Paasche price index as:

$$\widetilde{W}^{Paas}_{it,t-1} = \frac{\sum_{h \in \Omega^{m*}_{it}} W_{iht} M_{iht}}{\sum_{h \in \Omega^{m*}_{it}} W_{iht-1} M_{iht}} \tag{A25}$$

then the common-input expenditure ratio between t and $t-1$ is the product of the Laspeyres price index and the Paasche quantity index and also the product of the Laspeyres quantity index and the Paasche price index:

$$\frac{E^*_{it}}{E^*_{it-1}} = \frac{\sum_{h \in \Omega^{m*}_{it}} W_{iht} M_{iht}}{\sum_{h \in \Omega^{m*}_{it}} W_{iht-1} M_{iht-1}} = \widetilde{M}^{Lasp}_{it,t-1} \times \widetilde{W}^{Paas}_{it,t-1} = \widetilde{M}^{Paas}_{it,t-1} \times \widetilde{W}^{Lasp}_{it,t-1}$$

The definition of the alternative output quantity indexes is analogous to the definition of the input quantity indexes (A21), (A22) and (A23).

B Data Appendix

B.1 Manufacturing Survey

The *Encuesta Anual Manufacturera (EAM,* Annual Manufacturing Survey), carried out by the Colombian national statistical agency, *Departamento Nacional de Estadística* (DANE), can be considered a census of manufacturing plants with 10 or more employees. The sample includes plants with fewer employees but value of production above a certain level (which has changed over time). Also, once plants are in the survey, they typically are kept in the sample, even if employment or value of production fall below the cutoffs.

The survey distinguishes between the value of output produced and output sold (which may differ because of holding inventories) and the value of materials consumed and materials purchased. We use value of output produced and value of materials consumed and refer to these, with some looseness of language, as sales (or revenues) and material expenditures.

For each plant, we construct capital stock using the perpetual-inventory method with a depreciation rate of 0.05, using information only on machinery and equipment, including transportation equipment. That is, we calculate $K_{it} = K_{i,t-1} \times (0.95) + I_{i,t-1}$ where K_{it} is the capital stock of plant i in year t and $I_{i,t-1}$ is investment in machinery and equipment by plant i in year $t-1$. We set the initial value for each plant, K_{i0}, using the book value of machinery and equipment reported by the plant in its first year in the sample. We deflate both initial book value and investment by a price index for gross fixed capital formation calculated by Colombia's central bank. We sum capital stock across plants to get a firm-level measure.

DANE assigns plants to 4-digit industrial categories (International Standard Industrial Classification (ISIC) revision 2) in each year based on the sectors in which they have the most output. To each firm, we assign the 4-digit industry in which the firm has the most output over our study period, given DANE's plant-year-level assignments.

The EAM contains employment and wage-bill information for broad occupational categories and contractual status (permanent vs. temporary). Employment is average employment over the year, and the wage bill is the total wage bill for the year. The employment measure we use as a covariate is the total number of workers, including temporary workers. When calculating the average monthly earnings at the firm level (for use in comparing to the monthly minimum wage in the "bite" measure — see Subsection 2.4.1 in the main text), we use only permanent workers, since dividing annual earnings by twelve arguably gives a sensible measure of monthly earnings only for permanent workers, who have a higher likelihood of working 12 months per year.

B.2 Trade Data

The Colombian customs agency, *Dirección Nacional de Impuestos y Aduanas Nacionales* (DIAN), registers firm-level international trade transactions. Every registry corresponds to a purchase (import) or to a sale (export) by a Colombian firm and includes information on the date of the transaction, country of origin or destination, quantities purchased or sold, net weight of the shipment (in Kilograms) and total value of the transaction at the product 10 digits Harmonized System (HS) level. We exclude from our analysis the following: (1) Transactions with zero or negative total monetary value. (2) Transactions with zero or negative quantities. (3) Transactions with missing origin or destination. (4) Transactions made through a Free Trade Zone *(Zona Franca).* (5) Transactions of goods temporarily going out of the country for modifications and then coming back in. (6) Domestic transactions that are subject to taxes. (7) Transactions involving products corresponding to the HS 2-digit classifications: 27 (Mineral fuels, mineral oils and products of their distillation; bituminous substances; mineral waxes), 84 (Nuclear reactors, boilers, machinery and mechanical appliances; parts thereof) and 85 (Electrical machinery and equipment and parts thereof; sound recorders and reproducers, television image and sound recorders and reproducers, and parts and accessories of such articles). After making these exclusions we rank countries according to the total value of imports by Colombian firms for the period 1992-2009. We keep only transactions (imports or exports) between Colombian firms and foreign firms located in the top 100 countries of this ranking.

B.3 Household Survey Data

To construct the histogram of real wages in Appendix Figure A3, we use household surveys collected by DANE, the statistical agency. (In unreported results, we have constructed similar histograms by year for the entire 1992-2009 period.) We combine three different waves of surveys to compute monthly average wages at the individual level: *Encuesta Nacional de Hogares* (ENH) from 1992-Q2 to 2002-Q2, *Encuesta Continua de Hogares* (ECH) from 2002-Q3 to 2006-Q2 and the *Gran Encuesta Integrada de Hogares* (GEIH) from 2006-Q3 to 2009-Q4. When the survey reports daily or weekly wages we obtain monthly wages by multiplying the reported daily wage by 20.4 (approximate number of working days per month) or the reported weekly wage by 4.2 (approximate number of weeks per month). We restrict our analysis to wages reported by individuals employed by manufacturing firms with 11 or more workers and use the survey's individual sampling weights to compute the average monthly wage across locations and individuals.[5]

[5] In Colombia, in addition to the monthly minimum salary, employers are also required to pay a transport subsidy of approximately 9% of the minimum salary to workers who earn less than 2 times the minimum wage. The instructions in the household survey ask respondent not to include travel expenses *(viáticos)* in their wage reports. It appears that some respondents include the transport subsidy when reporting their wage and some do not; that appears to be why we see bunching in Appendix Figure A3 both at the minimum wage (203,826 nominal pesos, approximately 247,000 pesos in real terms (2000 pesos)) and at the minimum plus the transport subsidy (224,526 nominal pesos, approximately 272,000 pesos in real terms (2000 pesos)).

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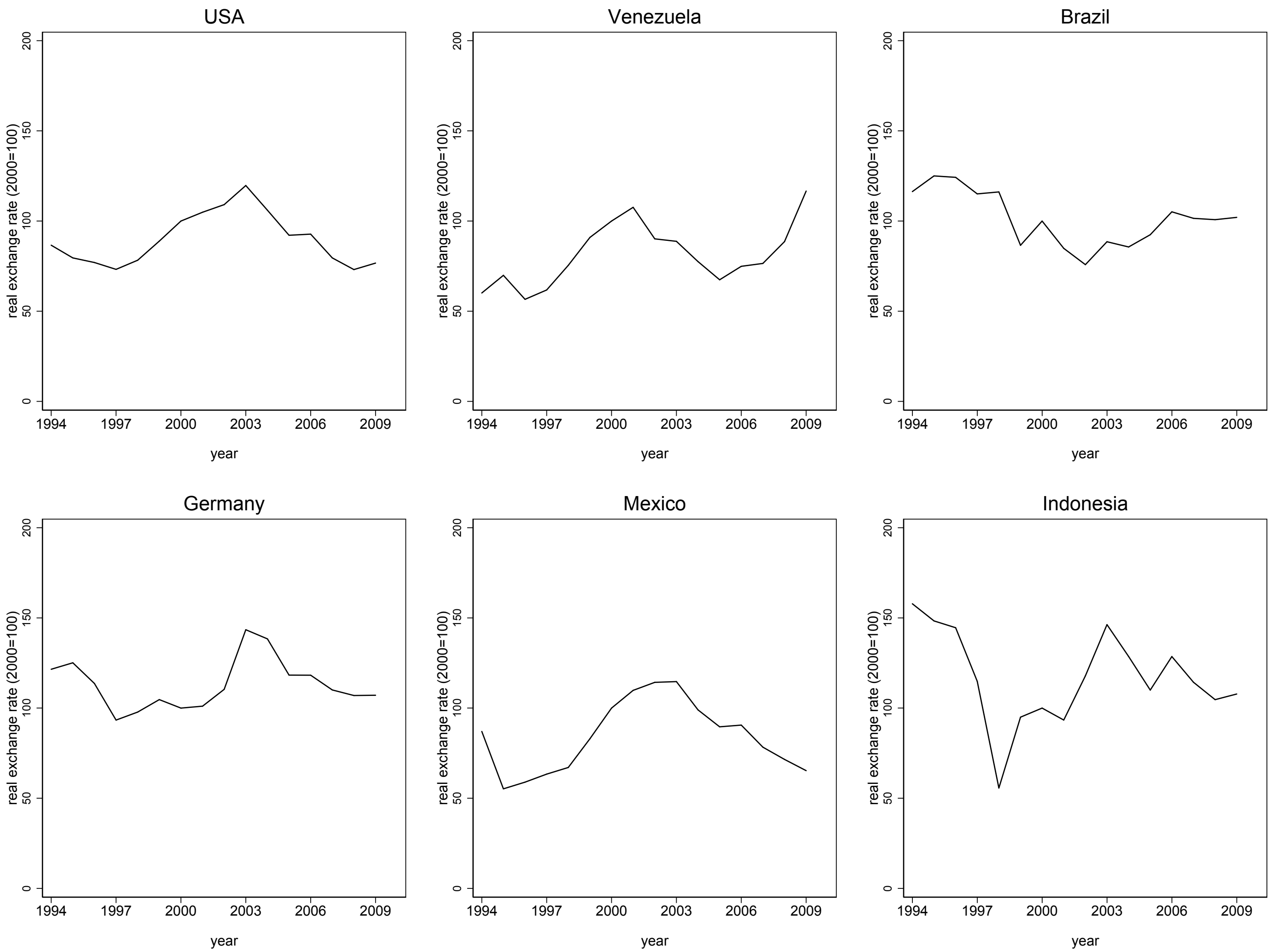
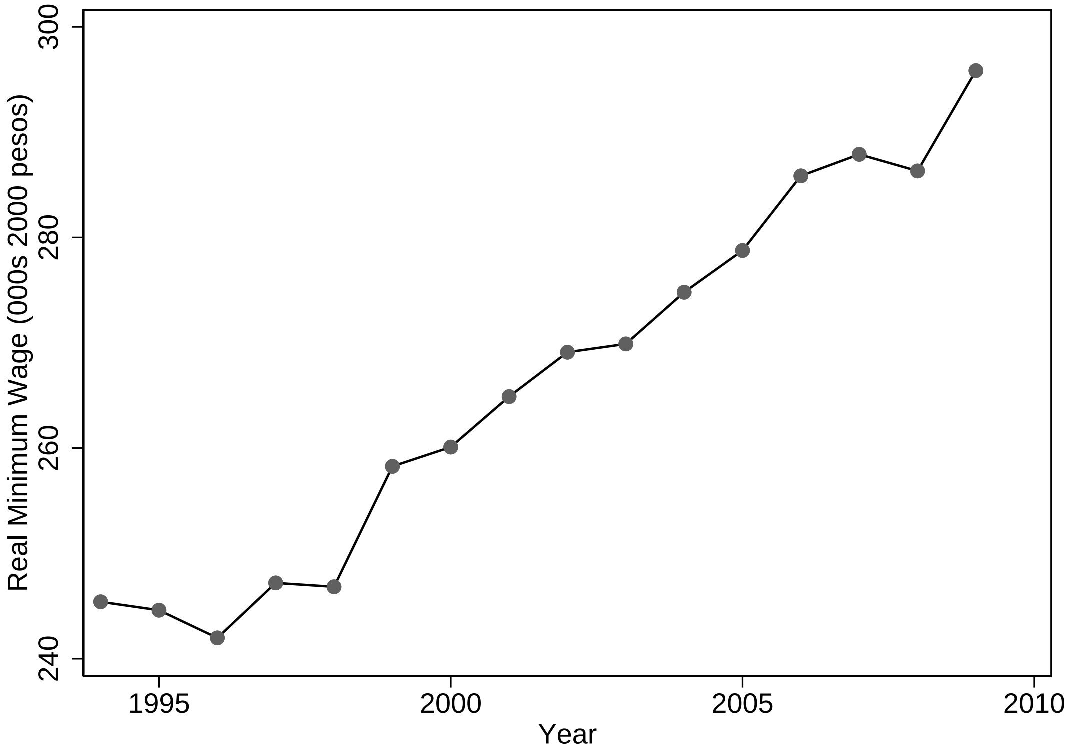
Figure A1. Real Exchange Rate Variation, 1994-2009



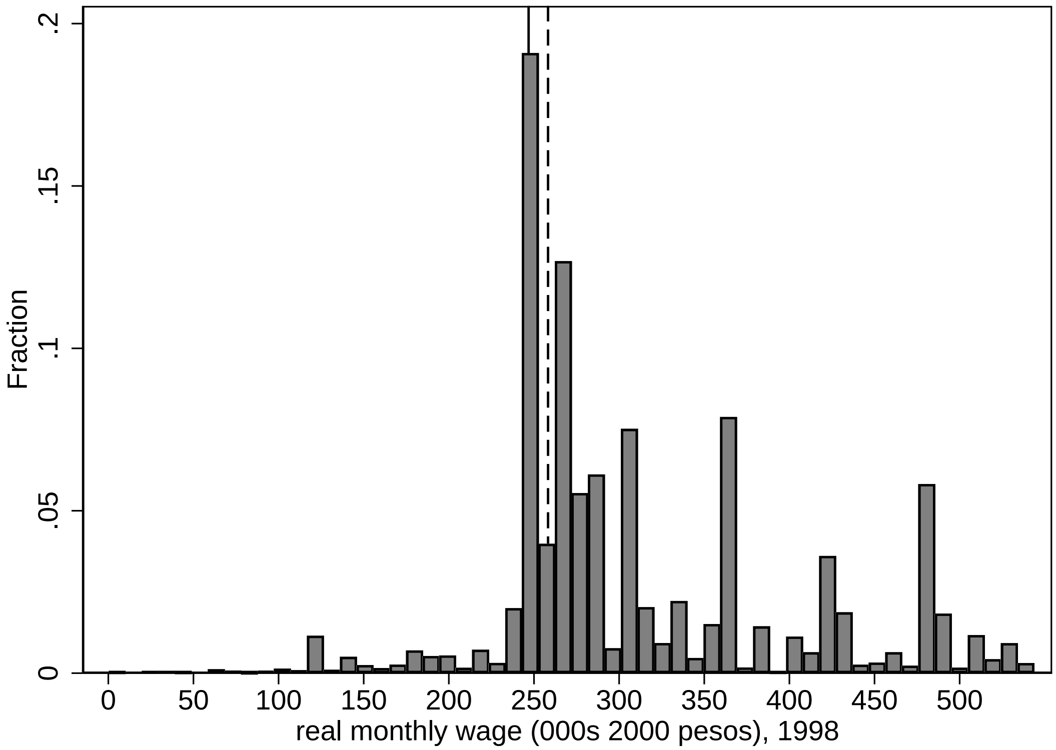
Notes: Figure plots real exchange rate (RER), normalized to 100 in 1994, calculated as in equation (17) in text, for top 6 import origins for rubber and plastics sectors. An RER increase reflects a real appreciation in the trading partner.

Figure A2. Real Minimum Monthly Wage, 1994-2009



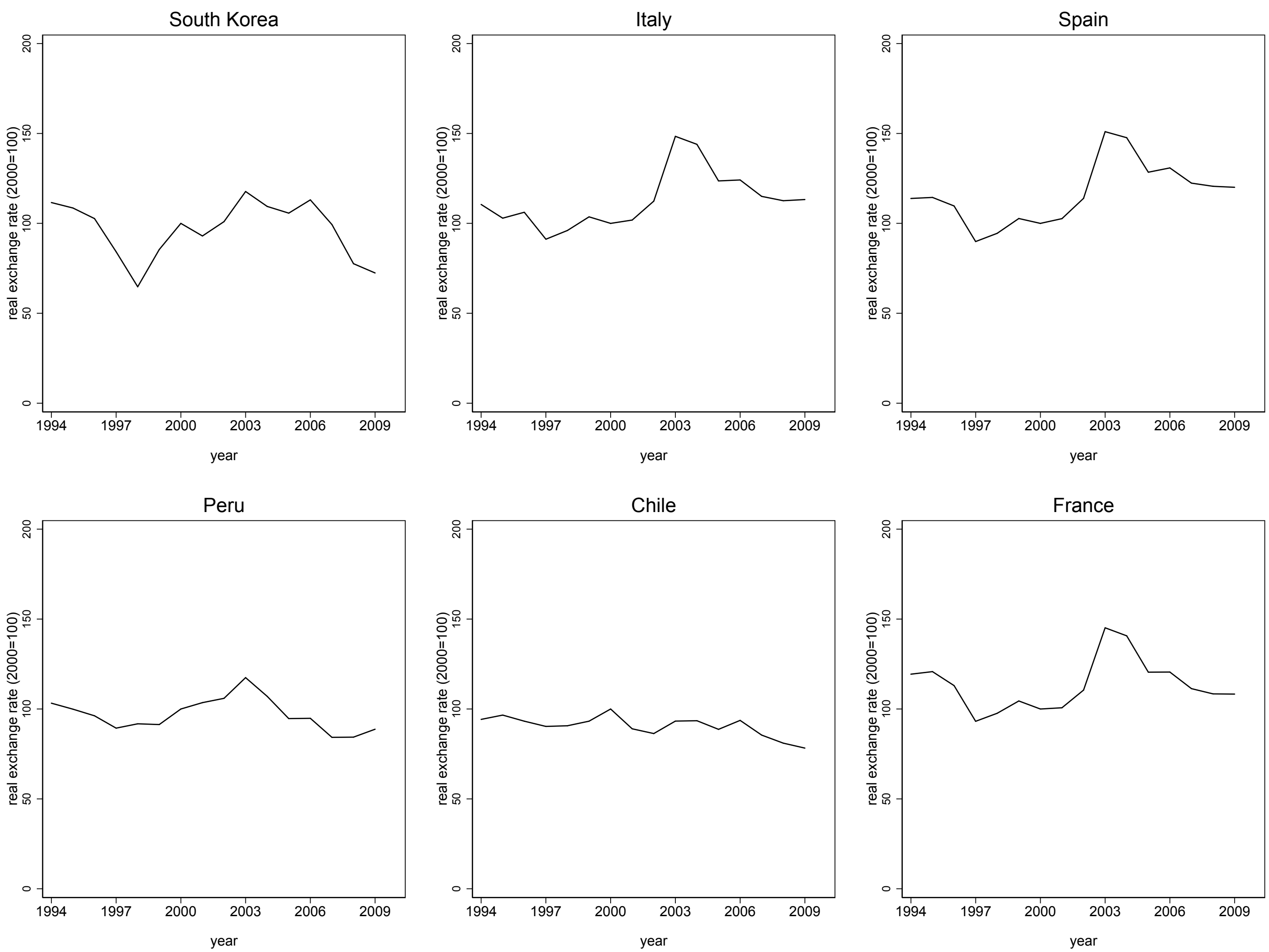
Notes: Figure plots Colombian national real monthly minimum wage, in thousands of 2000 pesos, for 1994-2009. Average 2000 exchange rate is approximately 2,000 pesos/USD.

Figure A3. Histogram of Real Wages from Household Survey, 1998



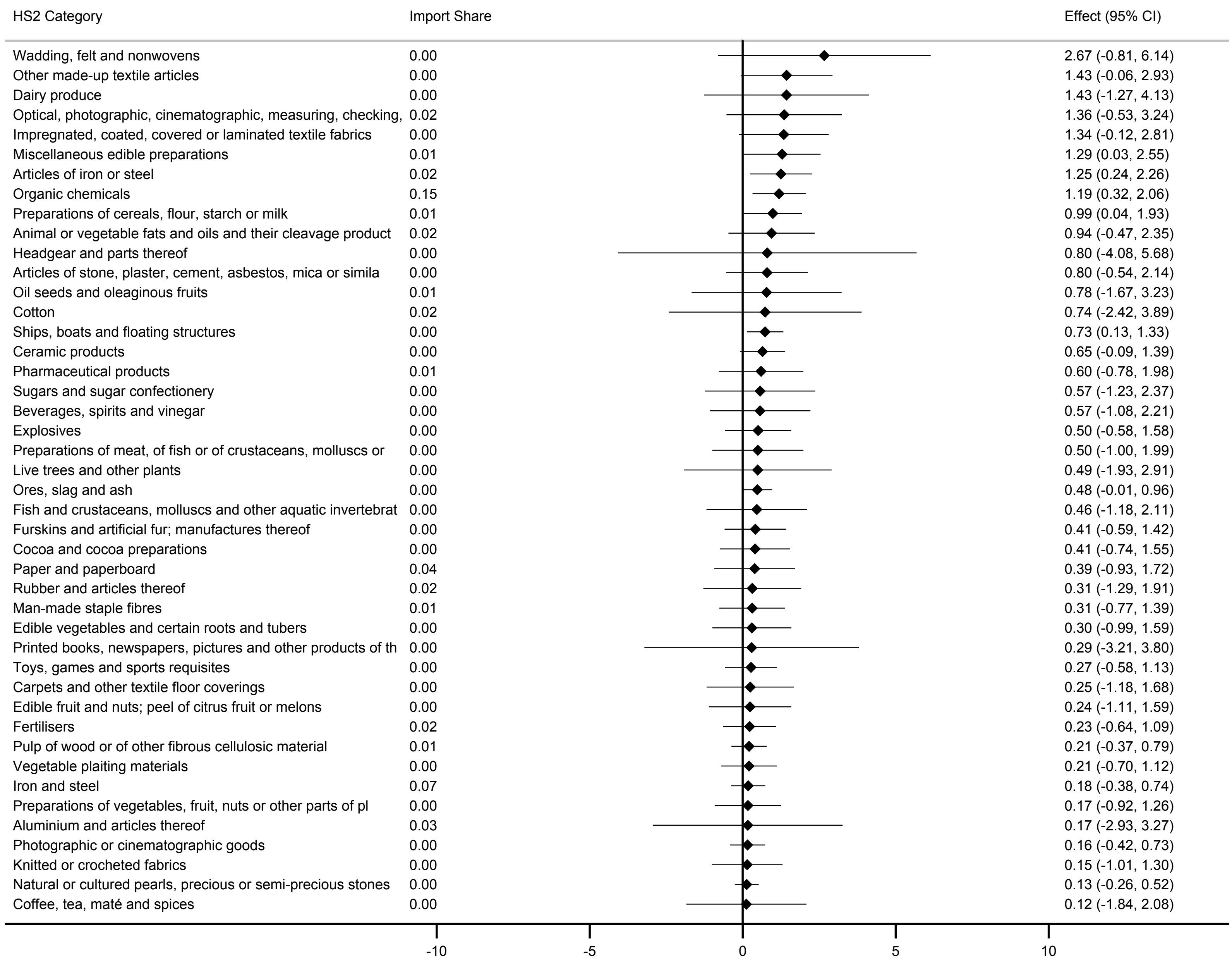
Notes: Histogram of real monthly wages in 1998, in thousands of 2000 pesos, from Encuesta Nacional de Hogares (ENH, National Household Survey). See Appendix B.3 for details. Bins are 10,000 pesos wide. Solid vertical line is national minimum wage in 1998, dashed vertical line is national minimum wage in 1999. Average 2000 exchange rate is approximately 2,000 pesos/USD.

Figure A3. Real Exchange Rate Variation, 1994-2009 (cont.)



Notes: Figure plots real exchange rate (RER), normalized to 100 in 1994, calculated as in equation (17) in text, for import origins ranked 7-12 for rubber and plastics sectors. (See Fig. A1 for ranks 1-6.) An RER increase reflects a real appreciation in the trading partner.

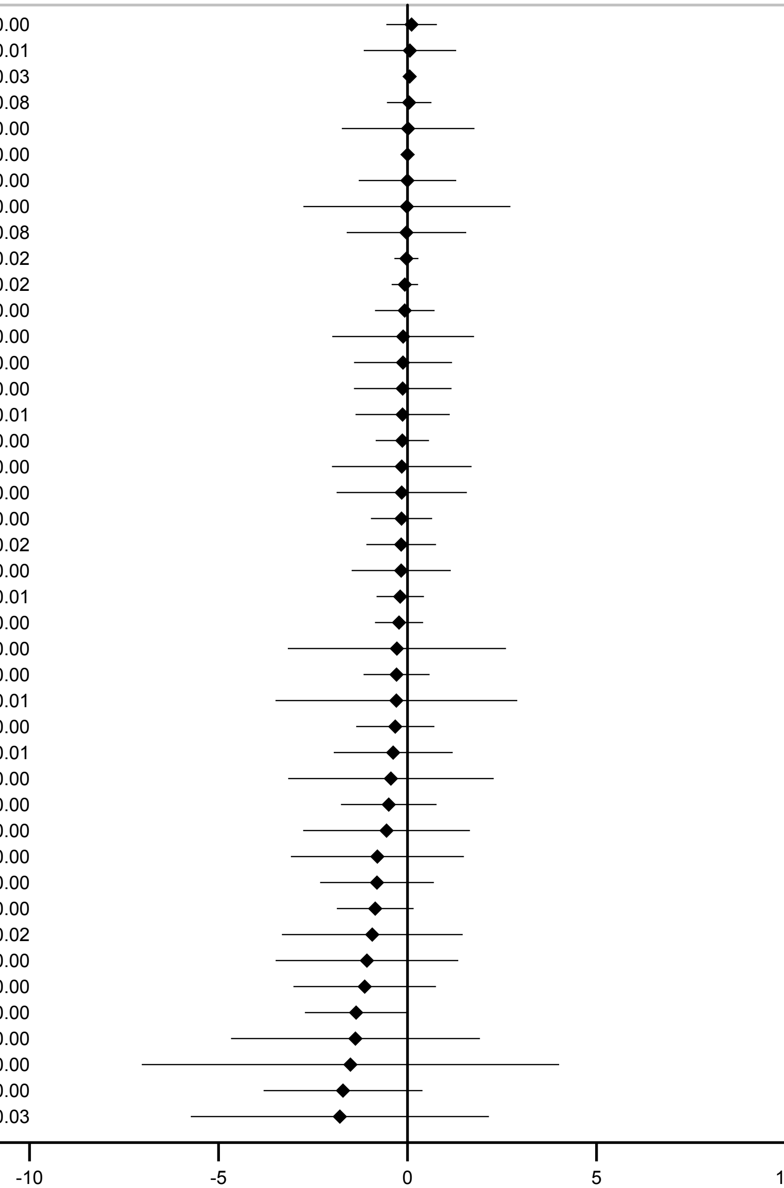
Figure A4. Coefficients from Import-Price Regressions



Notes: Average sector-specific coefficients from estimating equation (19) in main text. We generate 362 sets of leave-one-out coefficient estimates, then average both estimates and standard errors across firms and years. All Harmonized System 2-digit categories except except petroleum products, machinery and equipment (HS2 categories 27, 84 and 85) included; see Section 2.4.1 for details. Import share calculated as imports in HS2 category over total imports for 1994-2009 period.

Figure A4. Coefficients from Import-Price Regressions (cont.)

HS2 Category	Import Share	Effect (95% CI)
Railway or tramway locomotives, rolling stock and parts ther	0.00	0.11 (-0.56, 0.78)
Soap, organic surface-active agents, washing preparations, l	0.01	0.06 (-1.16, 1.29)
Miscellaneous chemical products	0.03	0.06 (-0.11, 0.23)
Plastics and articles thereof	0.08	0.04 (-0.54, 0.63)
Miscellaneous articles of base metal	0.00	0.02 (-1.74, 1.77)
Articles of apparel and clothing accessories, not knitted or	0.00	0.00 (-0.18, 0.18)
Silk	0.00	-0.00 (-1.29, 1.29)
Wood and articles of wood; wood charcoal	0.00	-0.02 (-2.75, 2.72)
Cereals	0.08	-0.03 (-1.61, 1.55)
Inorganic chemicals	0.02	-0.03 (-0.35, 0.29)
Essential oils and resinoids	0.02	-0.07 (-0.42, 0.28)
Other base metals; cermets; articles thereof	0.00	-0.07 (-0.86, 0.72)
Furniture	0.00	-0.12 (-1.99, 1.76)
Tools, implements, cutlery, spoons and forks, of base metal	0.00	-0.12 (-1.42, 1.18)
Products of the milling industry	0.00	-0.13 (-1.42, 1.16)
Glass and glassware	0.01	-0.13 (-1.38, 1.12)
Wool, fine or coarse animal hair	0.00	-0.14 (-0.84, 0.57)
Manufactures of straw, of esparto or of other plaiting mater	0.00	-0.15 (-2.00, 1.69)
Clocks and watches and parts thereof	0.00	-0.15 (-1.88, 1.57)
Nickel and articles thereof	0.00	-0.16 (-0.97, 0.65)
Tanning or dyeing extracts	0.02	-0.17 (-1.09, 0.75)
Special woven fabrics	0.00	-0.17 (-1.48, 1.14)
Man-made filaments	0.01	-0.19 (-0.82, 0.43)
Other vegetable textile fibres	0.00	-0.22 (-0.86, 0.41)
Works of art, collectors' pieces and antiques	0.00	-0.28 (-3.17, 2.61)
Tobacco and manufactured tobacco substitutes	0.00	-0.29 (-1.16, 0.58)
Zinc and articles thereof	0.01	-0.29 (-3.49, 2.90)
Articles of apparel and clothing accessories, knitted or cro	0.00	-0.32 (-1.36, 0.71)
Salt	0.01	-0.38 (-1.95, 1.19)
Lac; gums, resins and other vegetable saps and extracts	0.00	-0.44 (-3.16, 2.28)
Arms and ammunition; parts and accessories thereof	0.00	-0.50 (-1.76, 0.77)
Meat and edible meat offal	0.00	-0.56 (-2.76, 1.65)
Lead and articles thereof	0.00	-0.80 (-3.08, 1.49)
Raw hides and skins (other than furskins) and leather	0.00	-0.81 (-2.31, 0.70)
Prepared feathers and down and articles made of feathers or	0.00	-0.85 (-1.87, 0.16)
Copper and articles thereof	0.02	-0.93 (-3.32, 1.46)
Footwear, gaiters and the like; parts of such articles	0.00	-1.07 (-3.49, 1.34)
Tin and articles thereof	0.00	-1.13 (-3.02, 0.75)
Musical instruments; parts and accessories of such articles	0.00	-1.36 (-2.72, -0.00)
Live animals	0.00	-1.38 (-4.67, 1.91)
Umbrellas, sun umbrellas, walking sticks, seat-sticks, whips	0.00	-1.51 (-7.03, 4.01)
Albuminoidal substances; modified starches; glues; enzymes	0.00	-1.71 (-3.81, 0.40)
Residues and waste from the food industries	0.03	-1.79 (-5.73, 2.15)



Notes on first page of table.

Table A1. Primary Outputs and Inputs, Glass Products Producers

CPC code	Share of total revenues/expenditures	CPC description
A. Outputs		
3719102	0.21	Glass bottles for soft drinks
3719103	0.18	Glass bottles of a capacity not exceeding 1 liter
3711502	0.18	Safety glass
3711201	0.12	Unworked flat glass
3719104	0.11	Glass bottles of a capacity exceeding 1 liter
3711503	0.06	Safety glass for motor car, windshield glass and similar articles
3719101	0.03	Small glass jars for perfumery, pharmacy and laboratory
3712204	0.01	Glass wool sheet
3719309	0.01	Glass vases
2799704	0.01	Asphalt fabrics
4299942	0.01	Wire rods and rings, for brassieres
3719302	0.01	Glasswares of a kind used for table and kitchen
3712203	0.01	Fiberglass ducts
3719503	0.01	Glass ampoules
3712101	0.01	Fiberglass
3711601	0.01	Unframed mirror
3712907	0.01	Fiberglass bathtubs
3712908	0.01	Fiberglass tanks
3711501	0.00	Tempered glass
3719903	0.00	Glass screens
B. Inputs		
3711201	0.30	Unworked flat glass
3424501	0.22	Sodium carbonate
3711103	0.10	Waste and scrap of glass
3633019	0.07	Plastic fabric
3633007	0.05	Polyvinyl film
1531201	0.05	Siliceous sands and gravels
1639902	0.03	Feldspar
3219702	0.03	Printed labels
3474002	0.02	Polyester resins
1512004	0.02	Crushed or ground limestone
3215308	0.02	Partitions and dividers of carboard for boxes
3215302	0.01	Corrugated cardboard boxes
4151203	0.01	Angles, shapes and sections of copper
3511104	0.01	Anticorrosive bases and paints
3712101	0.01	Fiberglass
3170101	0.01	Wooden packaging box
4299942	0.01	Wire rods and rings, for brassieres
3170105	0.01	Pallets
3424202	0.01	Sodium sulfate
3641002	0.01	Unprinted plastic film in tubular form

Notes: Sample is producers of glass products (ISIC rev. 2 category 362). Shares calculated as revenues from output over total revenues (outputs) or expenditures on input over total expenditures (inputs) for 2000-2009 period, pooling firms and years.

Table A2. Summary Statistics, Glass Products

A. Period: 1996-2009

Number of Observations	410
Number of Firms	34
Number of Workers	122.97
Production value (billions 2000 pesos)	16.41
Earnings per year, permanent workers (millions 2000 pesos)	7.06

B. Period: 2000-2009

Input variables	
No. inputs per firm in average firm-year	9.43
Share of firms that import	0.65
No. inputs per firm in avg. firm-year, cond. on importing	9.50
Fraction of expenditure on imported inputs	0.46
No. imported HS8 categories in avg. firm-year, cond. on importing	24.74
Output variables	
No. outputs per firm in average firm-year	2.92
Share of firms that export	0.53
No. outputs per firm in avg. firm-year, cond. on exporting	3.48
Fraction of revenues from exported outputs	0.25
No. exported HS8 categories in avg. firm-year, cond. on exporting	5.31

Notes: Sample is producers of glass products (ISIC rev. 2 category 362). Exports and imports available in EAM data only in 2000-2009. Average 2000 exchange rate is approximately 2,000 pesos/USD.

Table A3. Difference Equation, Quantity Indexes, GMM-Style Instruments

	Dep. var.: $\triangle\ \widetilde{y}_{it}^{SV}$		
	(1)	(2)	(3)
$\triangle\ \widetilde{m}_{it}^{SV}$	0.580***	0.523***	0.434***
	(0.129)	(0.094)	(0.077)
$\triangle$ log labor ($\triangle\ell_{it}$)	0.442***	0.466***	0.441***
	(0.166)	(0.134)	(0.099)
$\triangle$ log capital ($\triangle k_{it}$)	-0.015	0.005	0.044
	(0.111)	(0.070)	(0.050)
N	4,247	4,247	4,247
Lag Limit	2	3	all
Number of excluded instruments	42	81	315
Hansen test	41.59	71.73	306
Hansen p-value	0.358	0.678	0.584
F - SW $\triangle\widetilde{m}_{it}^{SV}$	1.538	1.355	1.770
F - SW $\triangle$ log labor ($\triangle\ell_{it}$)l	2.186	1.797	1.814
F - SW $\triangle$ log capital ($\triangle k_{it}$)	3.324	3.100	2.328
KP LM statistic (underidentification)	50.78	104.4	387.9
KP Wald F-stat (weak instruments)	1.266	1.411	1.775

Notes: Table reports GMM estimation of our difference equation, (16), where further lags have been added "GMM-style" (Holtz-Eakin et al., 1988; Roodman, 2009), using only available lags and allowing separate coefficients in each period. Lags are included just to $t-2$ in Column 1, to $t-3$ in Column 2, and to the firm's initial year in Column 3. The first-stage coefficients are not reported, but number of instruments and the Sanderson-Windmeijer F-statistics corresponding to the first-stage regressions are reported in each column. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table A4. Differences (Step 1): First Stage, Alternative Aggregators

	$\triangle\widetilde{m}_{it}^{Torn}$ (1)	$\triangle\ell_{it}$ (2)	$\triangle k_{it}$ (3)	$\triangle\widetilde{m}_{it}^{Lasp}$ (4)	$\triangle\ell_{it}$ (5)	$\triangle k_{it}$ (6)	$\triangle\widetilde{m}_{it}^{Paas}$ (7)	$\triangle\ell_{it}$ (8)	$\triangle k_{it}$ (9)
$\widetilde{m}_{it-2}^{Torn}$	-0.015**	0.011***	0.026***						
	(0.006)	(0.004)	(0.005)						
$\widetilde{m}_{it-2}^{Lasp}$				-0.021***	0.012***	0.025***			
				(0.007)	(0.004)	(0.005)			
$\widetilde{m}_{it-2}^{Paas}$							-0.015**	0.013***	0.025***
							(0.006)	(0.004)	(0.005)
ℓ_{it-2}	0.012	-0.030***	0.042***	0.016	-0.031***	0.042***	0.010	-0.031***	0.043***
	(0.010)	(0.007)	(0.010)	(0.010)	(0.007)	(0.010)	(0.009)	(0.007)	(0.010)
k_{it-2}	0.005	0.008**	-0.049***	0.009	0.008**	-0.049***	0.005	0.008**	-0.049***
	(0.006)	(0.004)	(0.007)	(0.006)	(0.004)	(0.007)	(0.006)	(0.004)	(0.007)
$\triangle$ pred. import price index ($\triangle\widehat{\widetilde{w}}_{it}^{imp}$)	-0.210**	-0.047	0.118	-0.249**	-0.048	0.116	-0.260***	-0.046	0.120
	(0.099)	(0.063)	(0.102)	(0.098)	(0.063)	(0.102)	(0.099)	(0.063)	(0.102)
$\triangle$ log min. wage x "bite" ($\triangle z_{it}$)	-1.770**	-1.738***	-2.025***	-1.914**	-1.729***	-2.019***	-1.680*	-1.743***	-2.049***
	(0.885)	(0.495)	(0.600)	(0.860)	(0.496)	(0.599)	(0.871)	(0.495)	(0.600)
Year effects	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	4,247	4,247	4,247	4,247	4,247	4,247	4,247	4,247	4,247
R squared	0.024	0.038	0.042	0.027	0.038	0.041	0.026	0.039	0.041
F - statistic	3.209	6.860	14.496	4.337	7.164	13.538	3.095	7.143	14.035
F - SW	5.326	11.605	18.842	6.803	11.849	17.27	5.050	12.131	17.928
KP LM test (underidentification)		14.440			17.890			14.000	
KP Wald F-test (weak insts.)		3.010			3.830			2.927	

Notes: Specifications similar to Columns 7-9 of Table 5 but using alternative quantity indexes (Tornqvist, Lapeyres, Paasche) defined in Appendix A.4. Dependent variables at tops of columns. SW refers to Sanderson and Windmeijer (2016), KP to Kleibergen and Paap (2006). The F-statistic is the standard F for a test that the coefficients on the excluded instruments (indicated at left) are zero. The KP statistics, LM test for under-identification and Wald F test for weak instruments, are for each IV model as a whole, and are not specific to Columns 2, 5, 8. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table A5. Differences (Step 1): Second Stage, Alternative Aggregators

	△ log Tornqvist output index (1)	△ log Laspeyres output index (2)	△ log Paasche output index (3)
△ log Tornqvist materials index ($\triangle\widetilde{m}_{it}^{Torn}$)	0.470**		
	(0.206)		
△ log Laspeyres materials index ($\triangle\widetilde{m}_{it}^{Lasp}$)		0.400**	
		(0.166)	
△ log Paasche materials index ($\triangle\widetilde{m}_{it}^{Paas}$)			0.375*
			(0.195)
△ log labor ($\triangle\ell_{it}$)	0.414**	0.340*	0.434**
	(0.200)	(0.189)	(0.189)
△ log capital ($\triangle k_{it}$)	-0.203	-0.146	-0.216*
	(0.131)	(0.129)	(0.122)
Year effects	Y	Y	Y
N	4,247	4,247	4,247
R-squared	0.205	0.247	0.218
Materials Robust (LC) Conf. Interval 90%	[0.202 - 0.737]	[0.183 - 0.616]	[0.121 - 0.629]
Labor Robust (LC) Conf. Interval 90%	[0.154 - 0.674]	[0.095 - 0.586]	[0.189 - 0.679]
Materials Robust (LC) Conf. Interval 95%	[0.151 - 0.788]	[0.141 - 0.658]	[-0.129 - 0.678]
Labor Robust (LC) Conf. Interval 95%	[0.104 - 0.724]	[0.048 - 0.633]	[0.142 - 0.726]
Arellano-Bond AR(2) statistic	0.359	0.364	0.340
Arellano-Bond p-value	0.720	0.716	0.734

Notes: Specifications similar to Column 3 of Table 5 but using alternative quantity indexes as defined in Appendix A.4. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table A6. Levels (Step 2): Second Stage, Alternative Aggregators

	Dep. var.: $\widetilde{y}_{it} - \widehat{\beta}_m \widetilde{m}_{it} - \widehat{\beta}_\ell \ell_{it}$		
	Tornqvist	Laspeyres	Paasche
	(1)	(2)	(3)
log capital k_{it}	0.104	0.222	0.169
	(0.089)	(0.095)	(0.083)
	[0.209]	[0.188]	[0.197]
Year effects	Y	Y	Y
N	4,247	4,247	4,247
R-squared	0.081	0.192	0.146

Notes: The output and input aggregates used to construct the dependent variable are indicated at the top of each column. Columns correspond to Appendix Table A5. The first stage of this levels (Step 2) IV model is identical to that reported in Table 7. Uncorrected robust standard errors in parentheses. Corrected robust standard errors in brackets. See Section 2.4.2 for details. Tornqvist, Paasche and Laspeyres quantity indexes are defined in Appendix A.4. *10% level, **5% level, ***1% level.

Table A7. Differences (Step 1): First Stage, Including Export Price Index

	$\triangle \widetilde{m}_{it}^{SV}$	$\triangle \ell_{it}$	$\triangle k_{it}$
	(1)	(2)	(3)
$\widetilde{m}_{it-2}^{SV}$	-0.018***	0.013***	0.026***
	(0.006)	(0.004)	(0.005)
ℓ_{it-2}	0.012	-0.031***	0.042***
	(0.009)	(0.007)	(0.010)
k_{it-2}	0.007	0.008**	-0.049***
	(0.006)	(0.004)	(0.007)
$\triangle$ pred. import price index ($\triangle \widehat{\widetilde{w}}_{it}^{imp}$)	-0.254***	-0.056	0.134
	(0.099)	(0.063)	(0.102)
$\triangle$ log min. wage x "bite" ($\triangle z_{it}$)	-1.793**	-1.740***	-2.018***
	(0.862)	(0.495)	(0.601)
$\triangle$ pred. export price index ($\triangle \widehat{\widetilde{w}}_{it}^{exp}$)	0.007	0.103*	-0.163
	(0.094)	(0.062)	(0.100)
Year effects	Y	Y	Y
N	4,247	4,247	4,247
R-squared	0.026	0.039	0.042
F - statistic	3.611	7.437	13.640
F - SW	5.724	12.564	16.414
KP LM test (underidentification)		15.410	
KP Wald F-test (weak insts.)		3.248	

Notes: Dependent variables at tops of columns. SW refers to Sanderson and Windmeijer (2016), KP to Kleibergen and Paap (2006). The F-statistic is the standard F for a test that the coefficients on the excluded instruments (indicated at left) are zero. The KP statistics, LM test for under-identification and Wald F test for weak instruments, are for each IV model as a whole, and are not specific to Columns 2, 5, 8. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table A8. Differences (Step 1): Second Stage, Including Export Price Index

	Dep. var.: $\triangle$ log output index ($\triangle\widetilde{y}_{it}^{SV}$)
	(1)
$\triangle\widetilde{m}_{it}^{SV}$	0.375**
	(0.180)
$\triangle$ log labor ($\triangle\ell_{it}$)	0.395**
	(0.183)
$\triangle$ log capital ($\triangle k_{it}$)	-0.182
	(0.125)
$\triangle$ pred. export price index ($\triangle\widehat{\widetilde{w}}_{it}^{exp}$)	-0.040
	(0.095)
N	4,247
R-squared	0.238
Materials Robust (LC) Conf. Interval 90%	[0.141 - 0.609]
Labor Robust (LC) Conf. Interval 90%	[0.157 - 0.633]
Materials Robust (LC) Conf. Interval 95%	[-0.090 - 0.653]
Labor Robust (LC) Conf. Interval 95%	[0.112 - 0.679]
Arellano-Bond AR(2) statistic	0.338
Arellano-Bond p-value	0.735

Notes: Corresponding first-stage estimates are in Appendix Table A7. Robust standard errors in parentheses. Weak-instrument-robust confidence intervals are based on LC test of Andrews (2018), implemented by Stata twostepweakiv command. Arellano-Bond statistic and p-value test for serial correlation in residual, based on Arellano and Bond (1991). *10% level, **5% level, ***1% level.

Table A9. Differences (Step 1): First Stage, Alternative Samples

	plastics-only			including glass		
	$\triangle\widetilde{m}_{it}^{SV}$	$\triangle\ell_{it}$	$\triangle k_{it}$	$\triangle\widetilde{m}_{it}^{SV}$	$\triangle\ell_{it}$	$\triangle k_{it}$
	(1)	(2)	(3)	(4)	(5)	(6)
$\widetilde{m}_{it-2}^{SV}$	-0.022***	0.011***	0.026***	-0.020***	0.014***	0.027***
	(0.007)	(0.004)	(0.006)	(0.006)	(0.004)	(0.005)
ℓ_{it-2}	0.010	-0.031***	0.045***	0.011	-0.032***	0.041***
	(0.009)	(0.008)	(0.011)	(0.009)	(0.007)	(0.010)
k_{it-2}	0.011*	0.009**	-0.050***	0.010*	0.008**	-0.046***
	(0.006)	(0.004)	(0.007)	(0.005)	(0.003)	(0.006)
$\triangle$ pred. import price index ($\triangle\widehat{\widetilde{w}}_{it}^{imp}$)	-0.297***	-0.107	0.101	-0.235***	-0.039	0.099
	(0.106)	(0.067)	(0.114)	(0.079)	(0.049)	(0.078)
$\triangle$ log min. wage x "bite" ($\triangle z_{it}$)	-1.426	-1.641***	-2.161***	-1.692**	-1.644***	-1.816***
	(0.960)	(0.518)	(0.650)	(0.809)	(0.467)	(0.556)
Year effects	Y	Y	Y	Y	Y	Y
Observations	3,693	3,693	3,693	4,657	4,657	4,657
R-squared	0.030	0.037	0.041	0.028	0.040	0.041
F - statistic	3.847	6.147	11.922	4.626	8.235	15.575
F - SW	5.316	9.719	11.419	6.695	13.576	16.517
KP LM test (underidentification)		14.040			18.450	
KP Wald F-test (weak insts.)		2.958			3.852	

Notes: Table similar to Table 5, Columns 7-9, for alternative samples of (a) plastics producers only (Columns 1-3) and (b) rubber, plastic, and glass product producers (Columns 4-6). Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table A10. Differences (Step 1): Second Stage, Alternative Samples

	Dep. var.: $\triangle$ log output index ($\triangle\widetilde{y}_{it}^{SV}$)	
	plastics-only	including glass
	(1)	(2)
$\triangle$ log materials index ($\triangle\widetilde{m}_{it}^{SV}$)	0.351**	0.409**
	(0.175)	(0.170)
$\triangle$ log labor ($\triangle\ell_{it}$)	0.328*	0.439**
	(0.199)	(0.175)
$\triangle$ log capital ($\triangle k_{it}$)	-0.158	-0.188
	(0.123)	(0.122)
Year effects	Y	Y
Observations	3,693	4,657
R-squared	0.268	0.220
Materials Robust (LC) Conf. Interval 90%	[0.123 - 0.579]	[0.188 - 0.631]
Labor Robust (LC) Conf. Interval 90%	[0.070 - 0.587]	[0.212 - 0.666]
Materials Robust (LC) Conf. Interval 95%	[-0.102 - 0.623]	[0.146 - 0.673]
Labor Robust (LC) Conf. Interval 95%	[0.020 - 0.636]	[0.169 - 0.709]
Arellano-Bond AR(2) statistic	0.414	0.803
p-value Arellano-Bond test	0.679	0.422

Notes: Corresponding first-stage estimates are in Table A9. Samples are (a) plastics producers only (Column 1) and (b) rubber, plastic, and glass product producers (Column 2). Robust standard errors in parentheses. Weak-instrument-robust confidence intervals are based on LC test of Andrews (2018), implemented by Stata twostepweakiv command. Arellano-Bond statistic and p-value test for serial correlation in residual, based on Arellano and Bond (1991). *10% level, **5% level, ***1% level.

Table A11. Levels (Step 2): First & Second Stages, Alternative Samples

A. First stage

	Dep. var.: log capital (k_{it})	
	plastics-only	including glass
	(1)	(2)
$\triangle k_{it-1}$	0.616***	0.767***
	(0.110)	(0.110)
Year effects	Y	Y
N	3,693	4,657
R-squared	0.026	0.030
Kleibergen-Paap LM test	31.889	48.426
Kleibergen-Paap Wald F-test	31.409	43.404

B. Second stage

	Dep. var.: $\widetilde{y}_{it}^{SV} - \widehat{\beta}_m \widetilde{m}_{it}^{SV} - \widehat{\beta}_\ell \ell_{it}$	
	plastics-only	including glass
	(1)	(2)
log capital k_{it}	0.214	0.195
	(0.103)	(0.073)
	[0.204]	[0.180]
Year effects	Y	Y
N	3,693	4,657
R-squared	0.242	0.161

Notes: Uncorrected robust standard errors in parentheses. Corrected robust standard errors in brackets. See Section 2.4.2 for details. *10% level, **5% level, ***1% level.

Table A12. System GMM, Weak IV Diagnostics

Differences				Levels	
	Dep. var.: $\triangle$log sales$_{it}$				Dep. var.: log sales$_{it}$
Covariates	(1)	(2)	(3)	Covariates	(4)
$\triangle$log sales$_{it-1}$	0.264**	0.277***	0.183***	log sales$_{it-1}$	0.680***
	(0.106)	(0.091)	(0.060)		(0.108)
$\triangle$log expenditure$_{it}$	0.270**	0.387***	0.397***	log expenditure$_{it}$	0.556***
	(0.113)	(0.092)	(0.053)		(0.172)
$\triangle$log expenditure$_{it-1}$	-0.142*	-0.115*	-0.073	log expenditure$_{it-1}$	-0.274***
	(0.081)	(0.065)	(0.045)		(0.103)
$\triangle$ log labor ($\triangle\ell_{it}$)	0.290	0.339**	0.341***	log labor$_{it}$ (ℓ_{it})	-0.432*
	(0.183)	(0.144)	(0.070)		(0.239)
$\triangle$ log labor ($\triangle\ell_{it-1}$)	-0.077	0.069	0.046	log labor$_{it-1}$ (ℓ_{it-1})	0.481**
	(0.142)	(0.116)	(0.062)		(0.209)
$\triangle$ log capital ($\triangle k_{it}$)	0.009	-0.003	-0.004	log capital$_{it}$ (k_{it})	0.016
	(0.141)	(0.081)	(0.053)		(0.127)
$\triangle$ log capital ($\triangle k_{it-1}$)	-0.200*	-0.146*	-0.084*	log capital$_{it-1}$ (k_{it-1})	0.010
	(0.117)	(0.084)	(0.046)		(0.125)
N	4,247	4,247	4,247		4,247
R-squared	0.166	0.203	0.264		0.961
Lag Limit	2	3	all		NA
Number of excluded instruments	56	108	420		56
SW F-stat log sales$_{it}$	1.858	2.070	2.233		3.970
SW F-stat log expenditure$_{it}$	2.164	2.034	2.473		1.845
SW F-stat log expenditure$_{it-1}$	2.149	2.334	3.869		2.094
SW F-stat log labor (ℓ_{it})	1.796	1.643	1.985		1.238
SW F-stat log labor (ℓ_{it-1})	2.149	2.334	3.869		1.392
SW F-stat log capital (k_{it})	1.549	2.120	1.970		1.339
SW F-stat log capital (k_{it-1})	1.728	2.208	1.855		1.400
KP LM test (underidentification)	75.270	123.959	444.033		51.838
KP Wald test (weak instruments)	1.425	1.462	1.835		0.968

Notes: Table reports IV estimates corresponding to differences (Columns 1-3) and levels (Column 4) equations of System GMM, with weak-instrument diagnostic statistics. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table A13. System GMM, Using Sato-Vartia Quantity Indexes

	Dep. var.: log output index ($\widetilde{y}_{it}^{SV}$)		
	(1)	(2)	(3)
$\widetilde{y}_{it-1}^{SV}$	0.867***	0.862***	0.826***
	(0.031)	(0.030)	(0.027)
$\widetilde{m}_{it}^{SV}$	0.357***	0.398***	0.415***
	(0.082)	(0.078)	(0.055)
$\widetilde{m}_{it-1}^{SV}$	-0.306***	-0.325***	-0.297***
	(0.068)	(0.069)	(0.053)
log labor (ℓ_{it})	0.253*	0.200	0.217**
	(0.152)	(0.150)	(0.095)
log labor (ℓ_{it-1})	-0.204	-0.150	-0.183**
	(0.142)	(0.135)	(0.078)
log capital (k_{it})	0.225**	0.151*	0.101**
	(0.114)	(0.087)	(0.047)
log capital (k_{it-1})	-0.199**	-0.139*	-0.098**
	(0.099)	(0.076)	(0.043)
N	4,247	4,247	4,247
Lag limit	2	3	All
Hansen test	124.1	191.3	348.9
Hansen p-value	0.099	0.052	1.000

Notes: Table is similar to Table 8 but using CES output and input quantity indexes in places of log sales and log expenditures. The numbers of instruments are as indicated in Appendix Table A12. Robust standard errors in parentheses. *10% level, **5% level, ***1% level.

Table A14. Correlation of TFP Measures

	TSIV	OLS-SV	SysGMM	OP	LP	Wool-dridge	GNR	ACF
	(1)	(2)	(3)	(5)	(6)	(7)	(8)	(9)
TSIV	1.00							
OLS-SV	0.54	1.00						
SysGMM	0.94	0.77	1.00					
OP	0.34	0.92	0.61	1.00				
LP	0.45	0.95	0.70	0.99	1.00			
Wooldridge	0.76	0.92	0.92	0.87	0.92	1.00		
GNR	0.59	0.99	0.79	0.88	0.92	0.91	1.00	
ACF	0.38	0.79	0.57	0.80	0.81	0.75	0.77	1.00

Notes: Table reports pairwise correlation coefficients (using all available observations for each pair) of TFPR in levels defined as in equation (28) in footnote 59, using coefficient estimates as follows: baseline estimates from Table 6, Column 3 and Table 7, Panel B, Column 2 (TSIV); OLS using Sato-Vartia quantity indexes with year effects, as in Table 4, Panel B, Column 2 (OLS-SV); System GMM using all available lags, as in Table 8, Column 3 (SysGMM); Olley and Pakes (1996), Levinsohn and Petrin (2003), Wooldridge (2009), and Gandhi et al. (2020), as in Table 9 (OP, LP, W, and GNR, respectively); Ackerberg et al. (2015) using a value-added production function (ACF), estimated using Stata command prodest (Rovigatti and Mollisi, 2018).