Industrial Policy with Development Characteristics*

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Abstract

Fertilizer subsidies are one of the largest sector-specific policy instruments used by governments in developing countries. Fertilizer is also almost entirely imported, exposing it to international shocks. We study the design of time-consistent optimal fertilizer subsidy policy and how it responds to global shocks in a general equilibrium model with financial frictions, heterogeneous farming technologies, and sectoral labor choices. We quantify the model in Rwanda by collecting household-level data from 444 rural villages from 2020 - 2024 and use the Russian invasion of Ukraine – which caused the real price of fertilizer to double in Rwanda – to estimate key elasticities. The optimal subsidy falls by 90 percent on impact, reallocating production toward villages with less fertilizer-dependent technology. In the the data, the subsidy rises by 50 percent. The optimal transition path raises welfare by 0.5 percent in consumption-equivalent terms. Strategic government planning documents point to constraints imposed by the government's own development of the subsidy program as a rationale for this divergence and directly highlight an awareness of over-subsidization, as our model implies.

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1 Introduction

Governments all around the world use industrial policy to intervene in specific sectors. This type of sector-specific intervention requires confronting the reality of production in developing countries. Governments often operate with a limited set of policy instruments – targeting specific inputs or products even within an industry – and lack the ability to commit to a policy over time. The nature of production also differs from richer countries. Technology choice that prioritizes different input mixes interacts with pervasive market failures. The return to different input mixes also varies systematically across space, especially in dominant sectors like agriculture (Gollin et al., 2014), where local soil and weather characteristics play a critical role. Finally, important non-labor inputs are produced abroad, exposing these policies – and thereby the macroeconomy – to potential international shocks.

How, then, should a policymaker design optimal policy given the limited set of instruments at her disposal? We take this up in the context of fertilizer subsidies in Rwanda. Fertilizer subsides are perhaps the most pervasive form of industrial policy in agriculture – many Sub-Saharan African countries spend up to 25 percent of their agricultural budgets on them (Jayne et al., 2018; Holden, 2019) – and are often justified by low fertilizer use and the resulting productivity consequences (Restuccia et al., 2008; Boppart et al., 2023). Because fertilizer is almost entirely imported into developing countries, it is also at the forefront of recent normative policy debates. Russia's 2022 invasion of Ukraine caused global fertilizer prices to triple, causing fertilizer prices to rise in developing countries.[1] Our main quantitative exercise asks how a fertilizer subsidy should respond to a global price shock like the one observed in 2022.

We build a general equilibrium model that takes seriously the issues highlighted above. Our model is one of household-farmers subject to limited borrowing and financial frictions, similar to canonical models reviewed in Buera et al. (2015a) and Kaboski (2023) and applied to multi-sector environments in Buera et al. (2011) and Manysheva (2022). It has three core features. First, villages differ in their access to credit, which impacts their ability to finance agricultural production. Second, villages also differ in their technological returns to fertilizer and can choose between

[1] Russia's 2022 invasion of Ukraine tripled world fertilizer prices and prompted more than 700 policy responses worldwide (Amaglobeli et al., 2023) amid a renewed push to understand imported fertilizer's role in industrial policy. There is no shortage of views on this topic. In response to this shock, former World Bank president David Malpass called for a renewed focus on fertilizer industrial policy in Sub-Saharan Africa (Malpass, 2022). Former head of the U.N. World Food Programme David Beasley warned of "hell on earth" if fertilizer prices were not dealt with properly (Associated Press, 2022), echoing other arguments heard elsewhere (Shah, 2022). The African Development Bank called for "urgent countercyclical policy response such as subsidies to mitigate the impact of higher food and energy costs (African Development Bank Group, 2022)." Individual countries adjusted policy as well and we will discuss the Rwandan context in detail in this paper.

a traditional labor-only technology and a modern fertilizer-intensive one. The model therefore allows two rationales for low fertilizer use, both of which have widespread empirical support, but with different policy implications.[2] Finally, village production may not be perfectly substitutable in the consumption market.

These assumptions create scope for policy, driven primarily by the inability of equilibrium forces to generate productive reallocation in the face of financial frictions. A fertilizer price shock most severely affects villages with high technological returns to fertilizer. In an economy with perfect financial markets, equilibrium output prices respond to shift production toward less affected villages, as long as they produce sufficiently substitutable products. However, input choices no longer respond as strongly when financial frictions are present. That is, financial frictions decrease the output price elasticity of inputs. This eliminates the ability of equilibrium prices to generate the necessary reallocation and offers an efficiency rationale for policy: the planner uses the subsidy to induce it.

To quantify these forces we introduce a Ramsey planner who is subject to constraints relevant in a developing country context. She has only a fertilizer subsidy available funded via VAT tax and, consistent with Rwandan policy, cannot allow it to vary by village. While the subsidy is restricted from being place-based, a feature prominent in other forms of non-agricultural industrial policy (Kim et al., 2025), it will be place-*biased* due to the heterogeneity in both technological returns and financial frictions that interact with the specific input targeted. The variation in impact across different villages will determine quantitative magnitudes. Two features of the model govern this. The first is the ease with which farmers can substitute fertilizer for labor within the farm absent policy intervention. We allow extensive and intensive margin adjustments of this sort via a constant elasticity of substitution modern technology and a choice between it and a traditional, labor-only technology. Intuitively, this governs the sense in which a village can endogenously lower its own fertilizer share without government involvement. The second is the co-variation in technology and financial frictions. To the extent that shifting production to other more labor-intensive villages comes with a productivity cost, the planner will be less inclined to do so.

We measure these forces in the model by taking it to the data in Rwanda. We collect a household-level panel dataset from 2020 - 2024 for 15,000 households across 444 villages. In 2022, Russia invaded Ukraine and real fertilizer price in Rwanda doubled. This shock was the largest change in global fertilizer prices since the 1970s oil

[2] Marenya and Barrett (2009) and Theriault et al. (2018) show that soil characteristics affect the yield response to fertilizer even within small geographic regions of Kenya and Burkina Faso. Beaman et al. (2023) highlights large returns to agricultural credit in Mali. We provide additional evidence in Rwanda later in the paper.

crisis. Rwandan policy responded immediately. The government raised their subsidy from 30 to 45 percent and, for a sense of scale, caused the program to grow from 4 to 19 percent of the Agricultural Ministry budget between two seasons.[3] Taking the global shock and government response in tandem, Rwandan farmers paid 70 percent more per kilogram of fertilizer in 2022 than they did one season before. We show via professional price forecasts that this change was unexpected. This gives variation useful for identification along with the disaggregated data needed to study it.

In a reduced-form sense, we find statistically significant and economically meaningful changes in many dimensions. Farmers in villages that use more fertilizer at baseline decrease both fertilizer use and harvests relative to less exposed villages. At the same time, these high-fertilizer villages become relatively more labor intensive. Total wage earnings decline, driven entirely by declining earnings outside the village. Both output prices and wage changes are consistent with employment being pulled back into agriculture.

We then show that several key model parameters can be recovered from cross-equation restrictions implied by the model among these same shift-share coefficients. These include the CES aggregator for agricultural consumption, which governs the qualitative sign of the subsidy change, along with several other parameters that govern quantitative magnitudes, such as the CES substitutability between fertilizer and labor and the model-implied distortions that help us disentangle technology and financial differences. These results make use of the fact that casting the model in time differences offers a substantial simplification of the relationship between model and data, borrowing intuition from exact hat techniques used in trade (see Caliendo and Parro, 2022, for a recent review).

Armed with the calibrated model, our first quantitative exercise asks how optimal policy evolves when fertilizer price spikes, as it did in 2022. We find that the optimal subsidy falls from 6.6 to 1.0 percent on impact, an 85 percent decline. In contrast, the observed subsidy in Rwanda rises from 30 to 45 percent. We return to this gap shortly.

Optimal policy is driven by two key parameters. First, we find that villages produce substitutable products. Empirically, we come to this conclusion because we harvests fall while prices rise in fertilizer-intensive villages. Intuitively then, our economy is one in which farmers produce similar crops with different technologies and the planner's optimal strategy pushes production toward less affected, more labor-intensive villages when fertilizer prices rise. If we were to assume complementarity,

[3] These same patterns hold in several other countries. Burundi raised their subsidy from 40 to 60 percent in 2022. Ghana moved to eliminate their cocoa fertilizer subsidy in 2019, only to resume it in 2023. Kenya re-introduced a general fertilizer subsidy previously scrapped in 2020. Tanzania implemented an explicitly temporary fertilizer subsidy in 2022. Zambia held farmer prices fixed in 2022, implicitly raising the subsidy level.

as would be more natural in an economy of spatial production networks or sectoral comparative advantage, our results flip and the planner prefers to raise the subsidy.

The second important quantitative feature is the existence of an easily accessible labor-only technology. This allows the planner to incentivize labor-intensive production via a lower subsidy with a relatively small productivity cost. The key data moments here are that we see substantial movement on the extensive margin of fertilizer use in response to the shock, with limited corresponding declines in output. Interestingly, we also estimate nearly Cobb-Douglas modern production. This implies that the aggregate fertilizer elasticity is much larger than would be extrapolated from local variation in fertilizer use. These patterns are consistent with evidence of high aggregate elasticities found in cross-country data (Boppart et al., 2023), and we offer a micro-founded and policy-relevant interpretation of their importance.[4]

We then study the consequences of the optimal reform from the initial 30 percent steady state observed in the data, taking into account the transition path. Consumption-equivalent welfare increases 0.6 percent compared to the observed time path of the subsidy or by 0.5 percent compared to counterfactual policy of a constant 30 percent subsidy. These gains more than compensate for the existence of the shock. That is, given the choice, the average household would be willing to absorb the shock if it meant transitioning to the optimal subsidy instead of remaining at the baseline 30 percent subsidy with stable fertilizer prices.

However, maximizing average welfare has distributional consequences. Relative to the observed path, the optimal path causes welfare to rise by 0.9 percent among labor-intensive villages. And while 95 percent of villages gain from the optimal transition, the tail of the most fertilizer intensive villages see welfare fall by almost the same amount. A corollary of this result is that rationalizing the observed subsidy change in the data as optimal requires welfare weights that prioritize richer, more fertilizer-intensive farmers. A second is that any Rawlsian-like welfare function that prioritizes the poor, or uses the subsidy as a redistributive transfer, will not be able to rationalize the data for a simple reason: the poor do not use fertilizer. The planner would rather lower tax rates for the poor than subsidize an input they do not use.

We use recently-released strategic government planning documents to shed light on this result. In 2022, the government commissioned a series of consulting reports to investigate the elimination of subsidies due to their high cost. This alone suggests the government is aware it is over-subsidizing farmers, as our model predicts. But

[4] Other work, like Buera et al. (2011) and Midrigan and Xu (2014), also emphasizes the interaction between financial frictions and the extensive margin of technology, though primarily the inability of traditional firms to overcome the costs associated with the shift to modern production. The ubiquity of flows between the two in agriculture motivates our insurance-like interpretation. See also Dercon and Christiaensen (2011) and Suri (2011) for additional empirical evidence on high levels extensive margin switching in fertilizer.

moreover, they highlight how the program's evolution during a period of stable prices caused farmers to believe that the program was designed to keep prices stable, not as the long-term development program it was originally intended to be. While an irrelevant distinction for most of the program's existence due to stable global prices, it constrains the government's options in 2022. This implies that government is not valuing richer farmers *per se*, but is constrained by the same political economy constraints highlighted more broadly in industrial policy by Juhász and Lane (2024). In the Appendix we develop an alternative welfare function that includes these features and study its relative importance in the government's decision problem.

1.1 Related Literature

This paper relates to a number of different literatures. Most directly, it joins a literature focusing on the role of the agricultural sector in the development process, specifically related to the role of intermediate inputs (Restuccia et al., 2008; Boppart et al., 2023) and market failures in the sector like financial frictions (Manysheva, 2022). Several recent papers focus on understanding how fertilizer policy affects various outcomes (Diop, 2023; Garg and Saxena, 2023; Ghose et al., 2024). Most closely related in this dimension is interesting recent work by Mazur and Tetenyi (2024) and Chakraborty et al. (2025), both of whom study the impact of fertilizer subsidy programs (along with other policies) in general equilibrium models. Our focus on understanding how a subsidy should evolve with its corresponding input price is new to this literature and shows that financial frictions alone are not generally the key driver of the optimal subsidy price schedule.

Within this framework, we link our quantitative results to causal empirical estimates in a critical market for development policy, joining a growing methodological tradition in macro-development using exogenous variation to estimate parameters (Kaboski and Townsend, 2011; Buera et al., 2021; VanVuren, 2025). Unlike much of this work that uses "smaller scale" RCTs for estimation, the general equilibrium effects embedded in our empirics are central to our strategy linking model and data.

Our interest in the impulse response to an exogenous shock implies that our model is also closely related to models that study the implications of credit crunches in real financial friction models (Buera et al., 2015b; Buera and Moll, 2015), though our model is tailored in various ways to the specifics of our more developing-country implementation. We add to it a normative perspective on optimal policy away from steady state (as in Acemoglu et al., 2006; Itskhoki and Moll, 2019). We identify caveats and provide guidance on both the sign and magnitude of policy design in response to global input price shocks. Centrally, we highlight the importance of technological heterogeneity on qualitative and quantitative properties of optimal policy. These features of the model are more generally applicable than our particular implementation.

Finally, central to our results are how general equilibrium effects induce distributional consequences that are central to policy. On this point, see also Rotemberg (2019) for small-firm industrial policy and Falcao Bergquist et al. (2023) in relation to scaling small-scale agricultural inventions, albeit in an efficient model. A key point here is that these gains may not be enough to overcome the cost of financing the program, as highlighted in the education market by Fujimoto et al. (2023).

2 Setting, Data, and Fertilizer Price Shock

Global inorganic fertilizer production is concentrated in a small number of countries, because of the importance of comparative advantage in key inputs like natural gas. Russia is the largest exporter globally, implying that shocks to their ability to export play an important role in global price fluctuations. In this section, we detail how global fertilizer prices translated into changes in Rwandan policy. Rwanda offers a useful setting for several reasons related to the generalizability to other contexts. First, agricultural value added per worker is only half of GDP per capita, a labor productivity gap found in many other developing countries (Kruse et al., 2023; Herrendorf et al., 2022). Second, Rwanda imports the entirety of its inorganic fertilizer and the government is heavily involved in setting the subsidy level. Third, the policy responds directly to the shock described below. These same patterns hold in most developing countries.

2.1 Data

Our main data sources for the empirical results are primary data. We collect household data from 444 villages across most of Rwanda. This dataset was originally designed to measure the impact of an RCT on last-mile infrastructure and as such covers all of Rwanda except the relatively flat east part of the country.[5] The dataset began in 2020 with a baseline and has been collected annually since, implying 5 visits per household (2020 – 2024, inclusive). We exclude all treated villages from the RCT and focus exclusively on control villages. Despite our description as a household-level panel, these data are an individual-level panel. We collect household rosters and ask individual-level questions on demographics, earnings, and health and education

[5] See Macharia et al. (2022) for the pre-analysis plan and other details about the RCT.

outcomes, along with household-level questions about farming practices, savings, and other related decisions.

In the Appendix, we compare our data to nationally representative data from the 2020 Agricultural Household Survey (NISR, 2020) which is conducted every 3 years in Rwanda. Our households are similar in demographics and land-holdings, with an average farm size of 0.33 hectares in both. Our sample is slightly more likely to use fertilizer, though the slope with land size is equal across the two datasets.

2.2 Fertilizer Market Structure in Rwanda

The agricultural calendar in Rwanda is made up of two main cropping seasons. Season A begins with planting in September and ends with harvesting in January and February, while Season B runs from March to June. These seasons are generally referenced by the year of Season A's harvest, so that the 2022 season consists of Season A from September 2021 – February 2022 and Season B from March 2022 – June 2022.

Types of Fertilizer and Link to Global Markets The main fertilizers used in Rwanda all include nitrogen. These include DAP (a combination of nitrogen and phosphorus), a balanced nitrogen-phosphorus-potassium (NPK) 17-17-17, and urea (primarily nitogren). These three fertilizers make up 90 percent of the value of all fertilizers used in Rwanda in 2020 (National Institute of Statistics Rwanda, 2021). Nitrogen-based fertilizers use ammonia to deliver the nitrogen to plants, which is produced primarily with natural gas.[6] Synthesizing ammonia accounts for 3-5 percent of global natural gas consumption (Song et al., 2018). The centrality of natural gas in the production process implies two relevant features. First, shocks to natural gas prices will affect fertilizer prices. Second, countries with natural gas have a comparative advantage in production. Because of this, production is concentrated in a handful of countries and over 90 percent of demand in Sub-Saharan Africa – and 100 percent in Rwanda – is met via imports.[7]

Market Structure Fertilizer is not freely traded in Rwanda, instead managed by the Ministry of Agriculture and Animal Resources (MINAGRI). The government offers licenses for organizations to import fertilizer from abroad (usually 4 – 10 private firms and large NGOs), who then sell fertilizer via a set of licensed retailers scattered

[6] Ammonia (NH_3) is easier for plants to break down than the nitrogen in the air (N_2) due to the triple-covalent bond between the two nitrogen atoms. The Haber-Bosch process subjects natural gas to pressure, causing its hydrogen atoms to bind with nitrogen atoms in the air. The induced reaction, $N_2 + 3H_2 \rightleftharpoons 2\ NH_3$, creates the ammonia used in fertilizer.

[7] The only country that produces fertilizer domestically is Nigeria, which produces extremely small urea quantities. Interestingly, nearly all of this is exported off the continent.

throughout the country. The government sets two prices: the market price and the subsidy level. The market price is the price received by the importers from the national government, and roughly tracks international prices. Technically, importers sell to retail vendors at a subsidized rate and are reimbursed by the government. Farmers then pay the market price net of the subsidy.[8,9] The normal price-setting involves the government announcing prices on July 1 preceding fertilizer purchases for Season A, then holding it fixed for the crop year, resetting in the next July. We say "normal" here because we will shortly show how price announcements adjust after the 2021 fertilizer price shock.

Farmer Access to Subsidies By 2020, the fertilizer subsidy was universally available. Ndushabandi et al. (2018) and Spielman et al. (2023) offer detailed histories of the program, but we discuss some key points that motivate modeling decisions here.

The fertilizer subsidy program grew out of the Rwandan Crop Intensification Program started in 2007. At the start, the program was targeted explicitly. A farmer was required to grow at least one hectare of 6 priority crops and, in return, was given a coupon for up to two 50-kg fertilizer bags. By 2013, there was growing recognition that targeting was limiting take-up. Three large changes were put in place. First, additional crops were added so that every district in Rwanda had at least one crop covered. The subsidy now covers fertilizer for maize, beans, wheat, soybeans, rice, potatoes, cassava, bananas, vegetables and fruit trees, covering essentially all production in our dataset except coffee. Second, the mono-cropping requirement was eliminated in response to farmer concerns about diversification. Third, the coupon system was replaced with a flat subsidy.

Together, these changes implied that by 2015, nearly all farmers were eligible for subsidized fertilizer. There were no restrictions on farm size and the elimination of mono-cropping implied nearly all farmers were eligible. Over 90 percent of all fertilizer transactions in Rwanda now run through this subsidized system. Remaining ineligible farmers are mostly due to self-selection into not registering for the system. Anticipating the model, we will assume universal coverage of the subsidy for these reasons.

[8]This process is managed by the Rwandan government's Smart Nkunganire System (link here). There are no subsidized products sold outside this system which facilitates repayment.

[9]One natural question when dealing with non-market prices is the equilibrium possibility of quantity restrictions. In practice this seems to be of limited concern, as farmers can almost always access the fertilizer they demand. By law, any fertilizer importer is required to service all agro-dealers in the country. Thus, the fertilizer market is national instead of a collection of closed local fertilizer markets, limiting the possibility of local shocks driving a local mis-match of supply and demand. Second, even when there is the possibility of national under-supply, the government induces imports to correct the projected imbalance. Both our farmer survey evidence and discussions with importers and agro-dealers support the view that this margin is relatively unimportant, though could play an indirect role through the government's budget.

2.3 Shock to Global Fertilizer Prices and Response in Rwanda

The reliance on imports exposes countries like Rwanda to global shocks, especially those that affect countries like Russia. In 2020, Russia produced 32 percent of the world's NPK and 10 percent of urea, contributing 13 percent of all nitrogen exports in the world (FAOSTAT, 2024).

In 2022, the Russia-Ukraine war created two main issues that caused fertilizer prices to spike. First, fertilizer and ammonia exports declined, the latter of which made it difficult to process fertilizer in other, non-natural gas producing countries like Morocco. Recent estimates suggest Russian ammonia exports fell 63 percent between 2021 and 2022, and urea fell 23 percent (Glauber and Laborde, 2022). While Rwanda primarily imports fertilizer from Morocco and Saudi Arabia, the decline in global supply impacted global prices.[10]

The second implication of the war was its impact on the natural gas market. Russian natural gas and oil exports fell in 2022, with Europe attempting to substitute Russian oil and gas with U.S. natural gas after exiting a cold 2021 winter with limited gas reserves (EIA, 2023; Gil Terte, 2023). This redirected American gas exports toward Europe for mostly non-fertilizer uses. Thus, not only was fertilizer is relatively short supply globally, its most important input was increasing in price for non-agricultural reasons. In the Appendix, we show that monthly global fertilizer prices closely track natural gas prices.

The relative (but not total) importance of these shocks remains subject to debate, but clearly none are related to Rwandan policy decisions. Figure 1 shows they were also unanticipated. It plots the global prices of DAP and urea from 2015 to 2023 using the World Bank Commodity Price Outlook (World Bank, 2024). The nominal price shows the sharp rise from a period of relative stability, with DAP and urea rising to 2.5 and 3 times their 2020 level by 2022.[11] The dashed lines in Figure 1 plot the forecasted prices from each October release from 2018 to 2022. The 2018 - 2020 forecasts of continued stable prices were not realized.

[10] Morocco is the world's largest phosphate producer, thus an important country for DAP exports, which delivers nitrogen and phosphate. In 2020, 58 percent of Rwanda DAP imports came from Morocco, 28 percent from Saudi Arabia and 14 percent from Russia. By 2022, Russian DAP fell to zero and was partially made up for by an increase in Saudi Arabian fertilizer. The total quantity of Rwandan DAP imports fell from 24 million tons in 2020 to 15.6 million tons in 2022.

[11] The fertilizer price actually starts to rise in mid-2021. This was due to an issue of low U.S. coal inventories putting upward pressure on natural gas prices in 2021. That pressure was released at the end of 2021 on the backs of higher gas production and a mild winter (EIA, 2022). Thus, the proper non-war counterfactual would have been a temporary and relatively minor increase in natural gas prices (and thus fertilizer prices) in 2021 only. This is consistent with the 2021 forecast in Figure 1.

Figure 1: Global Fertilizer Prices and Projected Prices (Current USD/metric ton)

(a) DAP (b) Urea

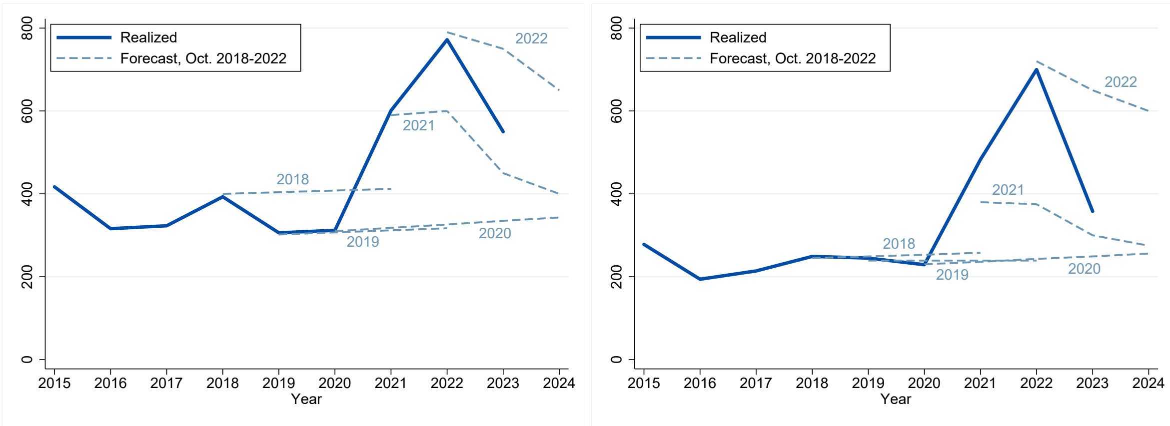

Figure Notes: The solid line plots the realized price for DAP and urea from 2015 - 2024. The dashed lines plot the projected prices each year. Both realized and projected prices are derived from the World Bank Commodity Price Outlook (World Bank, 2024).

2.3.1 Response in Rwanda

The Rwandan government historically updated fertilizer prices once per year. This policy-induced price stickiness was irrelevant when global fertilizer prices were stable during most of the 2000s. But higher global prices forced an unprecedented off-schedule price change in January 2022 before the start of Season B. The market price for DAP was adjusted upward by 40 percent. For some sense of scale, the average annual price change between 2018 and 2021 was 4 percent. Similar adjustments were made for urea and NPK.

In Figure 2, we use government documents to plot the government-defined market price for 3 main fertilizers along with the subsidy rate. Farmers pay (1-subsidy) × the market price. The figure also includes markers at any point when the government releases an updated price (even if that price is identical to the previous one). The discussion above is reflected in these figures. First, prices and subsidy rates remain stable until 2022. Second, through 2021, all price adjustments occur in Season A and are held fixed in B. However, in 2022-B, both the market price and subsidy rates are updated due to the stark rise in international prices.

The government made two adjustments in 2022. The first was an increase in the market price (Figure 2a). Second, they limited pass-through to farmers by increasing the subsidy rate (Figure 2b). As such, program jumped from 4 to 19 percent of MINAGRI's total budget; from 5 billion francs to 13 billion. Figure 2c then plots the implied nominal prices for farmers. To summarize, Figure 2d plots the relevant

Figure 2: Rwandan Fertilizer Prices

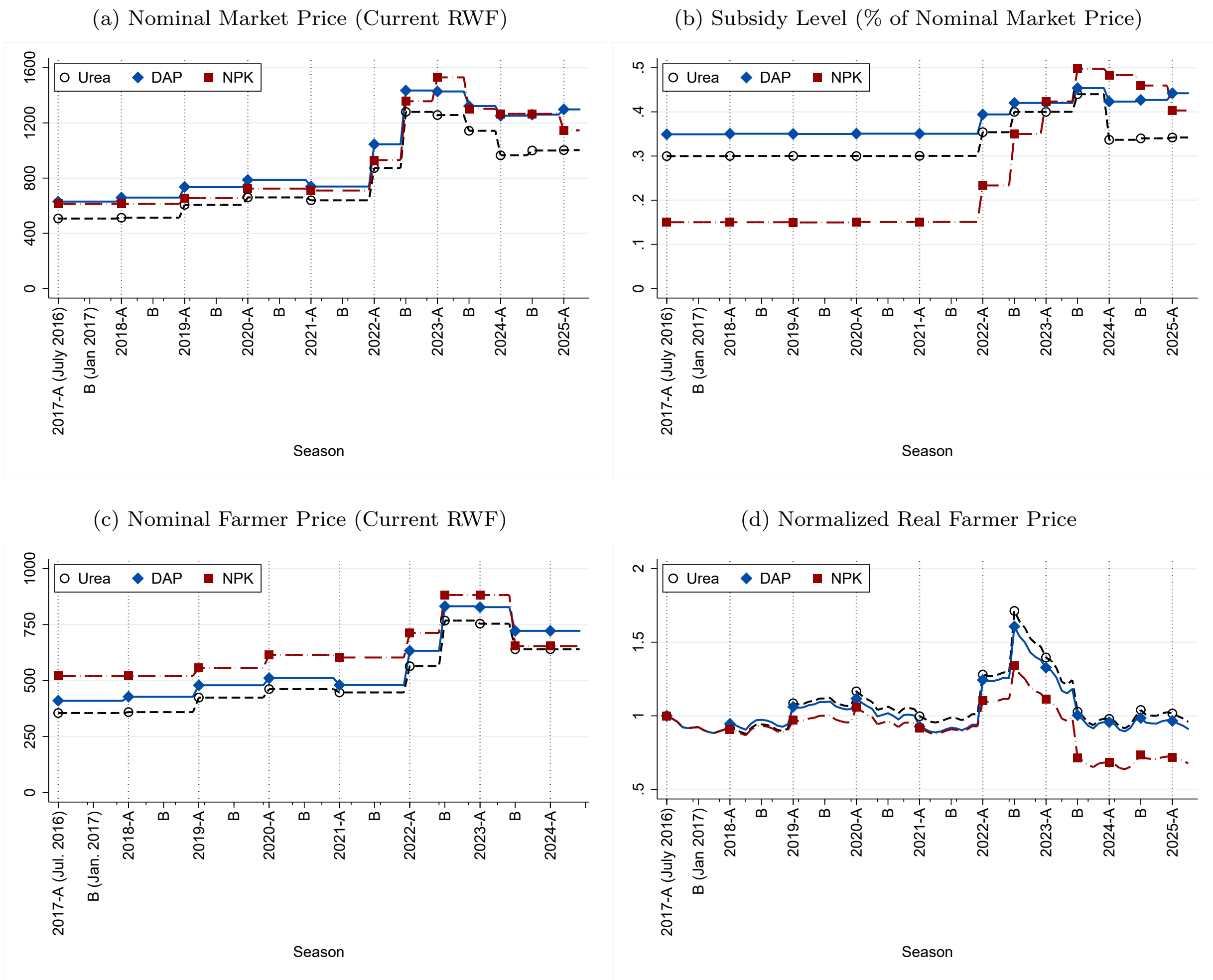

Figure Notes: Panel (a) plots the nominal market price defined by the Rwandan government in current Rwandan francs. Panel (b) plots the subsidy level. Panel (c) is the nominal farmer price, equal to (1-Subsidy) × Nominal Market Price. Panel (d) plots the nominal farmer fertilizer price deflated by the rural Rwandan CPI and normalizes by the 2017-A prices. In all figures, a marker is included when new prices are released even if the price is not updated. Notice that before 2022, all price updates were in Season A.

shock: the farmer fertilizer price deflated by the rural Rwandan CPI, normalized at 2017-A.

In the Appendix, we use data from traders to show that our data shows the same patterns (as should be expected, given that we survey the same retailers). Furthermore, we offer evidence that prices of other intermediate inputs, like fungicide and insecticide, do not show the same price increase.

3 Empirical Results

To estimate the impact of this shock across villages, we take advantage of the fact that villages differ substantially in their fertilizer intensity, measured here as total village fertilizer expenditure per acre of land in agricultural production in our 2020 baseline. We focus on village-level intensity instead of farmer-level intensity due to general equilibrium effects that will become clear below. The fertilizer intensity of village v at survey wave t is

$$\widetilde{f}_{vt} = \log\left(\frac{\sum_{h\in v} f_{hvt}}{\sum_{h\in v} \ell_{hvt}}\right).$$

where f_{hvt} is the fertilizer expenditures of household h in village v at year t. ℓ_{hvt} is land used in production. There is a substantial amount of variation in this moment at baseline, with a difference between fifth and ninety-fifth percentile village of 3.47 log points (from -1.75 to 1.72). We then consider the following regression

$$y_{hvmt} = \alpha + \beta Post_t + \eta\left(Post_t \times \tilde{f}_{v0}\right) + \delta_m + \theta_v + \zeta_t + \varepsilon_{hvmt}, \tag{3.1}$$

Here, y is the outcome for household h in village v in month m in year t. $Post_t$ equals one after January 2022, and we include village, month, and survey wave fixed effects.[12] Our interest here is the parameter η, which measures how outcomes vary with baseline fertilizer intensity $\widetilde{f}_{v0}$ after January 2022.

In all cases, we trim continuous outcomes at 1 percent and run the regressions as Poisson regressions to interpret as percentages with zeros. Outcomes in Rwandan francs are deflated by the monthly Rwandan CPI for rural consumers.

3.1 Income Sources

Table 1 covers the main income sources for households in the village. Panel A begins with farming outcomes.

Column (1) shows that fertilizer use declines more in places with higher baseline usage, showing that these villages are indeed more exposed to the shock (along with the mechanical aspect of the result that it bounded by zero below). Making use of the 3.47 log point 95/5 difference to get a sense of magnitude, the decline in fertilizer expenditures is about $-0.089 \times 3.47 = 31$ percentage points larger between the top and bottom of the distribution. Column (2) shows that much of this comes on the extensive margin, with the likelihood of using any fertilizer declining by 20 percentage points more at the top of the village fertilizer distribution. Thus, there is substantial

[12] Because of the number of villages covered by our study, survey waves are conducted over multiple months.

Table 1: Income Generating Activities

	Fertilizer		Harvest Value		
Panel A:	Expenditure	Use Any	Total	Cash Crops	Staple Crops
Agriculture	(1)	(2)	(3)	(4)	(5)
Post × Village Fertilizer Intensity	-0.089***	-0.059***	-0.060***	-0.050**	-0.065***
	(0.020)	(0.013)	(0.015)	(0.021)	(0.016)
Observations	40,070	40,070	40,456	40,456	40,456
Panel B:	Total	Outside Village	Inside Village		
Wage Earning	(6)	(7)	(8)		
Post × Village Fertilizer Intensity	-0.049***	-0.108***	0.014		
	(0.017)	(0.028)	(0.026)		
Observations	39,997	39,978	39,999		

Table notes: Standard errors clustered by village are in parentheses. The regression is given in (3.1) and is run as a Poisson regression. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and ***.

movement in the extensive margin of fertilizer use after the shock, a margin we will account for in our model. This decline in fertilizer impacts harvest. Column (3) shows that a one log point increase in baseline fertilizer intensity causes a 6 percent decline in harvests. Both cash and staple crops exhibit similar variation across villages.

Panel B covers total household labor market earnings, a second critical income source. Again, fertilizer intensive villages exhibit a much larger decline in wage income, entirely accounted for by a decline in wage earnings outside the village. Thus, the shock seems to draw workers back into a relatively less profitable farming sector.

3.2 Price Changes

The results above suggest a retrenchment of labor-intensive farm activity after the shock. We show that this is consistent with general equilibrium forces. We start with crop prices using a crop-household level regression

$$p_{chvmt} = \beta Post_t + \eta \left(Post_t \times \tilde{f}_{v0} \right) + \delta_{cm} + \theta_v + \zeta_t + \varepsilon_{chvmt},$$

where c is the crop and the remaining subscripts are as before, except we replace month fixed effects with crop-month fixed effects to take into account crop-specific seasonal price patterns. Table 2, Column (1) shows that the price of agricultural output rises after the shock.

Our second set of prices are wages. Here, defining $\mathbb{1}^{\text{inside}}_{ivmt} = 1$ if individual i's job

Table 2: Prices

VARIABLES	Crop Prices (1)	Daily Wage (2)
Post × Village Fertilizer Intensity	0.032***	–0.028**
	(0.006)	(0.011)
Post × Inside		–0.082***
		(0.017)
Post × Village Fertilizer Intensity × Inside		0.068***
		(0.0015)
Obs. Level	Crop-HH	Indiv
Observations	59,145	24,854

Table notes: Standard errors clustered by village are in parentheses. All outcome variables are in logs and regressions are run as OLS. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and ***.

is inside her village v in month m at wave t, we run the individual-level regression

$$w_{ivmt} = \alpha Post_t + \beta\Big[Post_t \times \mathbb{1}^{\text{inside}}_{ivmt}\Big] + \eta\left(Post_t \times \tilde{f}_{v0}\right) + \gamma\left(Post_t \times \tilde{f}_{v0} \times \mathbb{1}^{\text{inside}}_{ivmt}\right) + X_{ivmt} + \delta_m + \theta_v + \zeta_t + \varepsilon_{ivmt}$$

where w is the daily wage and controls include age, age squared, and indicators for education level (primary, secondary, vocational, tertiary). Thus, $\hat{\eta}$ measures the average effect and $\hat{\gamma}$ measures any differential effect for agricultural jobs in the village. These results are in Column (2) of Table 2. High fertilizer intensive villages see rising village wages relative to outside wages.[13]

To recap, villages using more baseline fertilizer (1) decrease fertilizer use and harvests, (2) see higher sales prices for crops, (3) see within-village agricultural wages rise, and (4) pull workers from other jobs into agricultural employment.

3.3 Additional Results in the Appendix

In the Appendix, we provide several additional results. First, we show that other intermediates like fungicide and insecticide do not change prices in the same way that fertilizer does. This acts as a placebo test because these intermediates are more readily available and produced in more countries and thus should not be subject to the same shock. Second, we show that the price effects observed here are driven by villages that are "large" in the markets in which they trade. Third, we use import and export data to show that Rwanda is importing primarily sunflower oil from Ukraine but little actual food, implying that a direct food supply shock is unlikely to drive these results. In total, Ukraine accounts for 0.6 percent of agricultural imports into Rwanda, almost all of which is sunflower oil. As a complement to this result, we

[13] While we do not ask households for a detailed price listing of non-agricultural purchases (only aggregated expenditures), we do collect those data from market traders. The two main non-agricultural products sold in local markets are cooking oil and salt. There is no change in the price of either. We similarly see no adjustment in fungicide prices.

use our own data to show that prices rise substantially on crops that are entirely produced and consumed locally in Rwanda suggesting an important role for domestic productivity.

3.4 "Exposure" to Shock and Underlying Mechanisms

The remaining issue here is that exposure – fertilizer expenditures per acre – is endogenous. It is driven by different underlying fundamentals that likely matter for policy. On one hand, if villages have heterogeneous technologies, high baseline fertilizer use is indicative of more exposure to the price shock in the usual sense. On the other hand, if villages instead differ in frictions like credit constraints, higher baseline fertilizer villages may be the least constrained and therefore most able to respond to the shock.

Left unanswered so far is which of these two broad sources drive baseline fertilizer use. The reality of production, especially in agriculture, is a combination of the two. Both are visible in our data. First, there is a strong correlation between fertilizer use and finance across villages in Rwanda. We use our baseline data across all villages to measure debt by village. Figure 3a plots a bin scatter of fertilizer expenditures as a share of harvest and its relationship to village debt. Figure 3b shows that higher debt correlates with walking distance to banks.

Figure 3: Fertilizer and External Finance

(a) External Debt and Fertilizer's Share of Harvest

(b) Average Walking Time to Bank

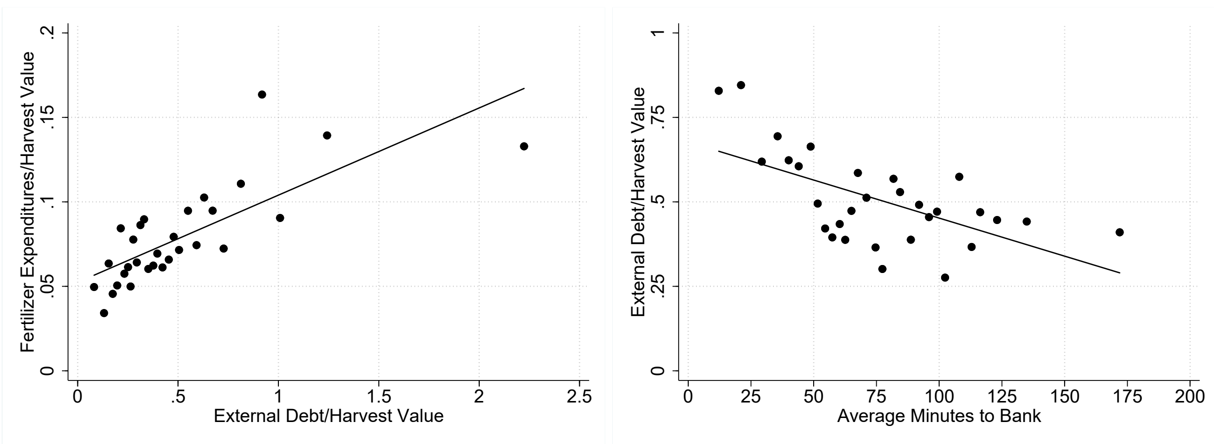

Figure Notes: Panel (a) plots fertilizer expenditures as a share of harvest value for all 444 villages and its relationship to total external debt held by the village. Panel (b) relates external finance levels to average walking time to the bank. Both sub-figures use bin scatters with 30 bins.

On the other hand, technology also varies systematically even within small geographic areas. Rwanda is a hilly country (especially in the west), which implies

Figure 4: Official Government Fertilizer Recommendation Zones for Potatoes

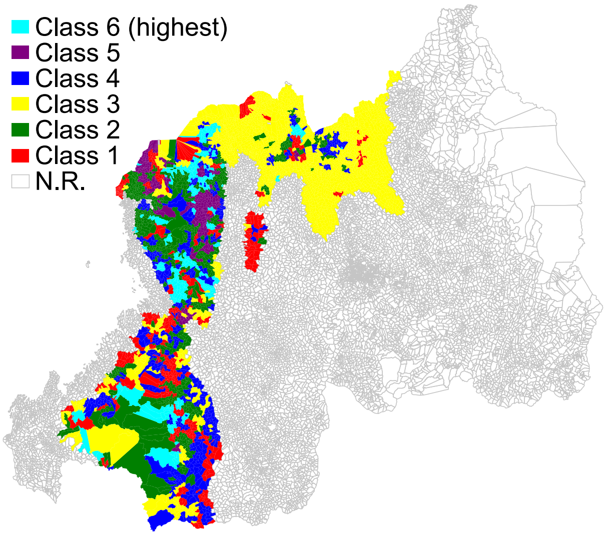

Figure Notes: This figure plots the 6 official fertilizer zone recommendations based off underlying soil characteristics and the combinations of DAP, NPK, and urea that are recommended by village. Roughly, the higher the class, the more intensive fertilizer use is required on the land. "N.R." stands for "not recommended."

substantial variation in soil quality – erosion, carbon, and nitrogen levels – that are critical inputs to the yield response to fertilizer.

Starting in 2018, the Rwandan government conducted thousands of soil samples around the country to build plot-level fertilizer recommendations for farmers. The Rwandan Soil Information System (RwaSIS) allows farmers a web portal to input their unique land identifier (UPI) and receive crop-specific fertilizer recommendations. We scrape this data from the website and plot the government-recommended fertilizer levels for potatoes which are grown in the western part of the Rwanda. Roughly, higher classes represent more intensive fertilizer per hectare recommendations.[14] Figure 4 shows that optimal fertilizer intensity varies even within relatively small geographic units, implying substantial heterogeneity in the technological returns to fertilizer across villages (see also Marenya and Barrett, 2009; Theriault et al., 2018, in Kenya and Burkina Faso on this point).

Household-Level Evidence These same patterns hold at the household level. We regress household-level fertilizer expenditures per hectare in 2020 on detailed soil characteristics (pH, nitrogen, organic carbon, and calcium) and bank distance. These soil characteristics are derived from the same detailed soil tests used as inputs into

[14]Since potatoes use primarily the balanced NPK 17-17-17 fertilizer blend, higher classes roughly represent more NPK and urea topdressing. The dataset itself comes from the Rwandan Soil Information System (RwaSIS). This online system offers farmers specific recommendations that they can access by inputting their unique land identifier (UPI), constructed in 2021 after 4 years of soil sampling across the country. We scrape this data from the website.

the optimal fertilizer recommendations of Figure 4, but have the benefit of being country-wide. We scrape these data from the web and link them to households in our data via GPS coordinates. We provide standard errors clustered at the village and the district level to account for potential spatial correlation in soil types (Conley (1999) standard errors generate the same conclusion; see the Appendix).

Table 3: Correlates of Fertilizer per Hectare use at the Household Level in 2020

	(1)
Log Minutes to Bank	-0.069 (0.031)** [0.058]
Carbon	
Slightly Low (2.51-3)	0.169*** (0.050)*** [0.087]*
Optimal (> 3)	0.659 (0.085)*** [0.145]***
Nitrogen	
Slightly Low (0.2-0.5)	-0.158 (0.062)** [0.104]
pH Level	
Low (5.01-5.5)	-0.227 (0.067)*** [0.133]
Slightly Low (5.51-6.0)	-0.790 (0.115)*** [0.231]***
Optimal (6.01-7.0)	-1.189 (0.128)*** [0.254]***
Calcium	
Low (500.01-1000)	0.094 (0.062) [0.095]
Slightly Low (1000.01-1600)	0.768 (0.115)*** [0.251]***
Optimal (1600.01-2400)	1.415 (0.105)*** [0.135]***
High (>2400)	1.637 (0.129)*** [0.227]***
Obs	8,581
R2	0.073

Table notes: Standard errors clustered by village and district are in parentheses and brackets. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***. Unlisted bins implies there is no soil realization of that type. For example, all soil in Rwanda is has nitrogen levels ≤ 0.5, so only includes the base "low" and the reported "slightly low" bins.

Table 3 shows two main results. First, bank distance is negative correlated with fertilizer use at the household level, as was true with the village-level patterns. Second, soil characteristics are also highly correlated with fertilizer expenditures. All Rwandan soil is low in nitrogen (all cells are either "low" or "slightly low"), but better soil is correlated with less need for fertilizer. Organic carbon is a critical input into the yield response to fertilizer, and indeed, high carbon levels are highly positively correlated with fertilizer expenditures. Finally, better pH balance is negative correlated with fertilizer use. Theses results show that this notion of technology is not simply capturing high versus low productivity land. Balanced pH soil is high quality soil, but requires less fertilizer to optimize, for example.

Together, these results characterize the reality of production in this environment: distorted firms make decisions in the presence of competitors with potentially substitutable products, but different input mixes due to technology differences, even absent frictions. Understanding the interplay between technology and frictions will matter for policy design. We next build a model to accomplish this task. We develop a model that takes seriously the co-existence of spatial variation in technology and financial frictions, and use the model to help disentangle their relative importance for quantifying optimal policy.

4 Model

The model is cast in discrete time where a time period is a year. We model the economy as a small open economy in which fertilizer is imported at the exogenous global price p_{xt}^{market}, consistent with what we observe empirically.

Sectors and Village Structure There are two sectors: non-agriculture, or "manufacturing" m for short, and agriculture a. Within the economy are a continuum of villages, denoted $j \in \mathcal{J}$. Each village contains a measure one of *ex ante* identical households that live forever, but will be heterogeneous *ex post* due to idiosyncratic productivity shocks and the corresponding savings decisions. Each household derives utility from each of the two sectors' final goods, and utility flow is Cobb-Douglas with discount rate β,

$$\sum_{t=0}^{\infty} \beta^t \Big[\zeta \log(c_{at}) + (1-\zeta)\log(c_{mt})\Big].$$

Households save s denominated in the agricultural final good with exogenous gross return R (which may be less than one due to storage costs or depreciation) and are restricted to savings, $s \geq 0$.

Villages differ in two ways: their technological fertilizer intensity α_j and the severity of their working capital constraint ϕ_j, both of which we will formalize below. The c.d.f. $G(\alpha, \phi)$ summarizes the distribution of villages across these two characteristics.

Manufacturing Production The manufacturing sector produces its final consumption good according to the production function $Y_{mt} = AH_{mt}$ where H_{mt} is efficiency units of labor hired in a centralized, competitive market that draws from all villages. We normalize the output price of the manufacturing sector to $p_m = 1$. In equilibrium, the wage per efficiency unit is $w_{mt} = A$ for all t.

Agricultural Production Each household i in village j produces a village-specific crop. They can use one of two technologies available to produce it. The modern production function is of the form,

$$y_{ij}^M = \frac{z_i}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}} \left(\alpha_j^{\frac{1}{\sigma}} x^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\gamma\sigma}{\sigma-1}}$$

where x is fertilizer and n is labor. α_j is sub-scripted, meaning that it differs across villages. All villages have the same returns to scale $\gamma \in (0, 1)$.

Input choices are made after the realization of z, a household-level productivity shock. We adopt a similar process to Buera et al. (2011), where each period with probability $1 - \psi$ a household receives a new draw $z_{i,t+1} \sim \text{Pareto}(\underline{z}_M, \theta_M)$ and with probability ψ, $z_{i,t+1} = z_{i,t}$. We summarize this c.d.f. as $Q(z_{t+1}, z_t)$. We scale productivity by a ratio $\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}$ to eliminate a mechanical productivity advantage that arises when α varies across villages.[15]

A household can alternatively choose to use the traditional technology,

$$y_{ij}^T = A_T n^\gamma.$$

The traditional technology uses only labor and no skill, and has productivity shifter A_T.

Regardless of technology, input choices are constrained by a working capital constraint that varies across villages,

$$p_x x + w_{aj} n \leq \phi_j s + \mathcal{W}_{ij}$$

where p_x is the price of fertilizer and w_{aj} is the village-specific equilibrium wage per unit of labor. This constraint says that household i in village j can leverage savings s to pay for inputs, up to a multiple ϕ_j, and similarly use payments to its endowment of labor services $\mathcal{W}_{ij}$ ($\mathcal{W}_{ij}$ will be defined shortly).[16] A household therefore chooses

[15] Anticipating our estimation, we find that modern production is Cobb-Douglas. An artifact of this is that villages near $\alpha = 0.5$ have a mechanical productivity advantage over places closer to $\alpha = 0$ or $\alpha = 1$. Our evidence discussed in Section 2 does not support this view. This re-scale eliminates it from the model as well.

[16] As a simple example, a household that uses a fraction n_o of their unit time endowment on their own farm faces a working capital constraint is $p_x x + w(n - n_o) \leq \phi s + w(1 - n_o)$, or $p_x x + wn \leq \phi s + w$. That is, the household funds payments to fertilizer and outside, hired labor.

the technology that maximizes profit subject to the working capital constraint

$$\begin{aligned}
\pi(s,z) &= \max\{\pi^T(s), \pi^M(s,z)\} \\
\text{subject to:} \quad \pi^T(s) &= \max_n \; p_{aj} A_T n^\gamma - w_{aj} n \\
\pi^M(s,z) &= \max_{n,x} \; p_{aj} z \left(\alpha_j^{\frac{1}{\sigma}} x^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\gamma\sigma}{\sigma-1}} - w_{aj} n - p_x x \\
p_x x + w_{aj} n &\leq \phi_j s + \mathcal{W}_{ij}
\end{aligned}$$

where p_{aj} is the relevant output price in village j. Overall, a village is defined by its technological fertilizer intensity α_j and the tightness of its working capital constraint ϕ_j.

Agricultural Consumption Good The village-specific crops are transformed into the final agricultural consumption good by a CES aggregator

$$Y_a = \left(\int_j y_j^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

so that a stand-in firm has the profit function

$$\max_{y_{aj}} \; p_{ac} \left(\int_j y_j^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} - \int_j p_{aj} y_{aj} dj.$$

Here, p_{ac} is the price of the final agricultural consumption good paid by consumers and p_{aj} is the output price received by households in village j. The continuum of households within any village implies no household market power for the differentiated products.

Household Labor Allocation Each worker in village j can work inside the village at the local market clearing wage w_{aj} or in the centralized manufacturing market for wage w_m. Individuals are identical in agriculture but receive draw ξ_m in manufacturing. The earnings of a worker are

$$\text{Earnings}(\xi_m) = \max\{w_a, \xi_m w_m\}$$

We assume throughout that ξ_m is distributed Pareto with minimum value 1 and shape θ. Because there are a continuum of individuals within a household, total household i labor earnings as

$$\mathcal{W}_{ij} = w_{aj} + \frac{w_{aj}^{1-\theta} w_m^\theta}{\theta - 1} \tag{4.1}$$

This is identical for all households in village j, so we drop the i subscript and write $\mathcal{W}_j$ going forward.

Fertilizer, Subsidies, and Government Budgets Fertilizer is imported and farmers pay a price p_x for it. p_x is made up of an exogenous market price p_x^{market} and the government subsidy τ_x, so that $p_x := (1-\tau_x)p_x^{\text{market}}$. The government buys at p_x^{market} and sells to farmers at $(1-\tau_x)p_x^{\text{market}}$ incurring a cost of $\tau_x p_x^{\text{market}}$ on each unit.

That subsidy is financed by taxes, subject to a balanced budget constraint, where government revenue is collected via a VAT tax on non-agricultural consumption.[17] The government earns τ_m on each unit given the normalized manufacturing output price.

Household Problem in Village j The timing of the model works as follows. At $t-1$, households choose savings s_t to bring into period t. Some of that depreciates they enter t with Rs_t. Then the shock z_t is realized. Households then make decisions on farm inputs and the allocation of labor, income is realized, and consumption and savings decisions are made.

The individual state of the household is therefore (z, s), and the aggregate state is the distribution μ across individual states and villages along with the fertilizer price and tax system $\boldsymbol{\tau} := (p_x^{\text{market}}, \tau_x, \tau_m)$. The household knows the full sequence $\{\boldsymbol{\tau}_t\}_{t=0}^{\infty}$ when making decisions. Recursively, the problem is

$$
\begin{aligned}
& v_j(z,s,\mu,\boldsymbol{\tau}) \;=\; \max_{c_a,c_m,s',\psi\in\{0,1\}} \; \zeta\log(c_a) + (1-\zeta)\log(c_m) + \beta\int_{z'} v_j(z',s',\mu',\boldsymbol{\tau}')dQ(z',z) \\
s.t. \quad & p_{ac}c_a + (1+\tau_m)c_m + p_{ac}(s'-Rs) = \max\{\pi^M,\pi^T\} + \mathcal{W}_j \\
& \pi^M = \max_{x,n} \; p_{aj}\left(\frac{z}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}}\right)\left(\alpha_j^{\frac{1}{\sigma}}x^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}}n^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\gamma\sigma}{\sigma-1}} - w_{aj}n - (1-\tau_x)p_x^{\text{market}}x \\
& \pi^T = \max_{n} \; p_{aj}A_T n^{\gamma} - w_{aj}n \\
& (1-\tau_x)p_x^{\text{market}}x + w_{aj}n \le \phi_j s + \mathcal{W}_j \\
& s' \ge 0
\end{aligned}
$$

The first constraint is the budget constraint, the next two define profit for both

[17]This tax system is consistent with that faced by rural households in developing countries. In Rwanda, expenditures on agricultural inputs and outputs are exempt from VAT. And while there is in principle income taxes on agricultural profits, the thresholds exempt nearly all non-commercial-scale farming. Farm incomes must be above 12 million RWF (9,200 USD in 2024, about 9 times GDP per capita) to incur any tax burden. Other taxes are designed to capture revenue from the agricultural sector, but play a minor role. For example, market fees are paid by traders who operate stalls in markets, but these are small revenue generators and we exclude them for simplicity. Similarly, non-agricultural earnings are subject to income taxes (above a threshold), but given that much of this work is informal in our setting, we refrain from modeling this tax on non-agricultural wages.

technologies, the fourth is the financial friction, and the fifth is the no-borrowing constraint. We define the decision rule $\psi(s, z, \mu, \boldsymbol{\tau}) = 1$ if the household chooses the modern farming technology.

We note here that when we introduce a shock to the fertilizer price p_x^{market}, we start the economy at its stationary equilibrium with baseline price p_{x0}^{market} (motivated by the stability of prices and subsidies in Section 2), then unexpectedly shock it with a deterministic sequence of prices $\{p_{xt}^{\text{market}}\}_{t=1}^{t=\infty}$. Hence, the recursive program above includes perfect foresight of the evolution of the aggregate state μ.

4.1 Recursive Competitive Equilibrium

Given a sequence $\{\tau_{xt}, p_{xt}^{\text{market}}\}$, a recursive competitive equilibrium of this model is a set of decision rules for households $(c_a, c_{mj}, x_j^M, n_{aj}^M, n_{aj}^T, s_j', \psi)$, the manufacturing firm N_m, and the final agricultural good producer y_{aj}, prices $\{p_{aj}\}, p_{ac}, w_a$ and tax rate τ_m such that (i) the household's decision rules are consistent with its optimization problem given prices and taxes, (ii) N_m solves the manufacturing firm's decision problem, (iii), y_{aj} solves the agricultural final goods firm decision problem, (iv) the law of motion for the aggregate state, $\mu' := \Lambda(\mu)$, is consistent with the decision rules, and (v) the government balances its budget and markets clear:

1. Government budget balance:

$$\tau_x p_x^{market} \int_j \int_{s,z} x_j(s,z) d\mu_j dj = \tau_m \int_j \int_{s,z} c_{mj}(s,z) d\mu_j dj$$

2. Market clearing:

   (a) Agricultural market: for each j,

$$\begin{aligned} y_{aj} &= \int_{s,z} \psi(s,z) z \left(\alpha_j^{\frac{1}{\sigma}} x_j(s,z)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_j)^{\frac{1}{\sigma}} n_j(s,z)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\gamma\sigma}{\sigma-1}} \mu_j + \\ &+ \int_{s,z} (1-\psi(s,z)) A_T n_j^T(s,z)^{\gamma} d\mu_j \end{aligned}$$

   (b) Agricultural final goods market:

$$\left(\int_j y_{aj}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} = \int_j \int_{(s,z)} c_{aj}(s,z) d\mu_j dj$$

   (c) The agricultural labor market: for each j,

$$\int_{s,z} n_j(s,z) d\mu_j = 1 - \left(\frac{w_m}{w_{aj}} \right)^{\theta}$$

(d) The manufacturing labor market:

$$H_m = \frac{\theta}{\theta - 1} \int_j \left(\frac{w_m}{w_{aj}} \right)^{\theta - 1} dj$$

A stationary equilibrium involves a constant sequence for subsidies and fertilizer market prices, $\{\tau_{xt}, p_{xt}^{market}\} = (\tau_{x,ss}, p_{x,ss}^{\text{market}})$ for all t and an aggregate state μ^* such that $\mu^* = \Lambda(\mu^*)$.

4.1.1 Ramsey Planner

The recursive competitive equilibrium above takes as given $\{\tau_{xt}, p_{xt}^{\text{market}}\}$. The planner then chooses a sequence of subsidies $\{\tau_{xt}\}$ given the market price sequence $\{p_{xt}^{\text{market}}\}$. We assume she cannot commit to future policy.[18] Therefore, our notion of equilibrium is a Markov-perfect equilibrium, in which the planner has no profitable deviations available. The planner maximizes

$$W(\mu_0, \{\tau_{xt}\}) = \int_{z,s,j} v_j(z, s, \mu_0, \boldsymbol{\tau}) d\mu_0(z, s, j)$$

subject to the resulting recursive competitive equilibrium and the condition that there are no welfare-improving deviations from $\{\tau_{xt}\}$ at any period t. A stationary Markov equilibrium is a recursive competitive equilibrium such that when $\mu_0 = \mu^*$ and $p_{xt}^{\text{market}} = p_{x,ss}^{\text{market}}$ for all t, the planner chooses a value τ_x^* such that $\tau_x^* = \text{argmax } W(\mu^*, \tau_x^*)$ with a similar no deviation condition.

4.2 Analytic Results

Our ultimate goal here is to understand how the optimal subsidy τ_x should respond to change in the market price of fertilizer p_x^{market}. The main economics of this problem are based on the interaction of financial frictions and technological heterogeneity. Other features – like the agricultural technology choice – play an important quantitative role, as we will show in the coming sections. To highlight the key trade-offs facing the planner, we characterize a simpler special case of our model that will look familiar to readers of standard macro-development models with an agricultural sector, such as Restuccia et al. (2008).

To that end, we make the model static, which requires a small change in the

[18] Policy with commitment is not time consistent in this model. Therefore, any exercise in which we start the economy from its stationary equilibrium, then allow the planner to adjust policy, will lead to optimal subsidy adjustments even in the absence of price changes. That is, our quantitative exercise both changes the price and breaks commitment. In addition to its realism, assuming no commitment implies that the planner would never adjust the subsidy in the absence of a price change. We discuss this more in the quantitative results.

financial friction to remove savings,

$$p_x x + w_a n \leq \phi_j + \mathcal{W}_j$$

where now ϕ_j is an exogenous parameter in village j instead of a multiple of savings. It also makes any notion of planner commitment irrelevant. We make two additional assumptions. First, we assume a Cobb-Douglas production function for the modern sector and remove the traditional sector. Village j therefore has the production function $y_j = x^{\alpha_j} n^{\eta_j}$, where $\eta_j = \gamma - \alpha_j$. We will show in the quantitative results that our data point us toward Cobb-Douglas production. We remove idiosyncratic productivity as well, but this is purely notational and has no bearing on the results here. Second, we assume away skills in non-agricultural labor, so that each individual chooses $\max\{w_{aj}, w_m\}$ leading to the result that $w_{aj} = w_m = A$ for all villages j.[19]

In this simplified model, a household in village j solves

$$\begin{aligned}
&\max_{c_a, c_m, x, n_a} \quad \zeta \log(c_a) + (1-\zeta)\log(c_m) \\
s.t. \quad & p_{ac} c_a + (1+\tau_m) c_m = p_{aj} x^{\alpha_j} n^{\eta_j} - (1-\tau_x) p_x^{\text{market}} x - wn + w \\
& (1-\tau_x) p_x^{\text{market}} x + wn \leq \phi_j + w
\end{aligned}$$

We define the competitive equilibrium of this simplified economy in the Appendix, though it differs only slightly from the one defined above.

4.3 Characterizing the Frictionless Equilibrium

To understand how τ_x should respond to fertilizer price changes, it is helpful to start from the frictionless economy with $\phi_j = \infty$ for all villages. Here, $\tau_x = 0$, so $p_x = p_x^{\text{market}}$. Characterizing this economy allows us to show how equilibrium forces work to reallocate production across heterogeneous villages. We then use this as a benchmark for the economy with binding financial frictions.

Because we are interested in changes between two equilibria, it turns out to be much simpler to characterize outcomes in differences instead of levels. To start, Lemma 1 characterizes the new equilibrium (with higher p_x^{market}) in terms of the baseline. All proofs are in the Appendix.

[19] A common way to break this equivalence is to add an exogenous tax on non-agricultural wages, so that $w_a = (1-\tau)w_m$ (e.g. Restuccia et al., 2008). Adding such a term does not affect our results in this section.

Lemma 1. *Assume that the market fertilizer price rises to $p_{x2}^{market} > p_{x1}^{market}$. Define*

$$\mathcal{Y} = \frac{\int_j y_{j1}^{\frac{\nu-1}{\nu}} dj}{\int_j \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{j1}^{\frac{\nu-1}{\nu}} dj}.$$

This value $\mathcal{Y}$, combined with exogenous parameters, fully characterizes the p_{x2}^{market} frictionless equilibrium relative to the p_{x1}^{market} equilibrium. For example, the equilibrium price ratios are

$$\frac{p_{ac2}}{p_{ac1}} = \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma}{\nu-1}} \qquad \textit{and} \qquad \frac{p_{aj2}}{p_{aj1}} = \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma-1}{\nu-1}} \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}}$$

The term $\mathcal{Y}$ determines the ratio of baseline aggregate output an adjusted version of itself, where the adjustment depends on the change to the fertilizer price p_x.[20] Thus, it summarizes the aggregate decline in production from the higher fertilizer price. The key feature of $\mathcal{Y}$ is that it depends on no endogenous values from the new p_{x2} equilibrium so that it fully characterizes the new equilibrium in terms of the baseline one.

α_j affects how village j's output price p_{aj2}/p_{aj1} changes after the shock. This alone is not surprising: villages have different fertilizer cost shares in production, and thus are affected differently. But *where* the new equilibrium wants to redistribute lost production is at the heart of this problem. To make this more explicit, we combine Lemma 1 with a bit of algebra to characterize the change in household farm profit (we derive this explicitly in the proof of Lemma 1),

$$\pi_{j2} = \underbrace{\mathcal{Y}\left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}}}_{\equiv \text{ adjustment to } \pi_{j1}} \underbrace{(1-\gamma)p_{j1}y_{j1}}_{\equiv \pi_{j1}} \tag{4.2}$$

which implies that profit rises in village j if and only if

$$\pi_{j2} > \pi_{j1} \Leftrightarrow \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma}{(\nu-1)(1-\gamma)}} > \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{1-\gamma}}.$$

The right-hand side is the partial equilibrium decline in village j profit from the higher fertilizer price. It is the change in the fertilizer price modulated by the fertilizer profit share, implying higher losses for villages with higher α. The left-hand side is the equilibrium change in revenue. This GE response depends not on α_j, but on

[20] In the proof of Lemma 1, we show that $Y_{a2}/Y_{a1} = 1/\mathcal{Y}^{\frac{\nu}{\nu-1}-\gamma}$.

how that village production is aggregated, summarized by the parameter ν. Thus, understanding the total effect requires understanding how the direct, PE effect of the shock compares to the GE price response across villages. Lemma 1 lets us characterize these effects. Re-writing (4.2) in log differences (and recalling that $p_x = p_x^{\text{market}}$ here) gives

$$\Delta \log(\pi_j) = \log(\mathcal{Y}) + \frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}\Delta \log(p_x^{\text{market}})$$

or as a derivative measuring its response to a fertilizer price shock,

$$\frac{\partial \Delta \log(\pi_j)}{\partial \Delta \log(p_x^{\text{market}})} = \frac{\partial \log(\mathcal{Y})}{\partial \Delta \log(p_x^{\text{market}})} + \frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}.$$

This leads immediately to a summary measure of how profit changes across all villages after a fertilizer price shock. We specify this in Proposition 1.

Proposition 1. *In the frictionless economy, the cross-village variation in equilibrium profit after a change in the market fertilizer price is*

$$\frac{\partial^2 \Delta_t \log(\pi_j)}{\partial \Delta_t \log(p_x^{market})\partial \alpha_j} = \frac{(1-\nu)}{\nu-(\nu-1)\gamma}\begin{cases} < 0, & \text{if } \nu > 1 \\ = 0, & \text{if } \nu = 1 \\ > 0, & \text{if } \nu < 1 \end{cases}$$

for any non-degenerate distribution of α.

Proposition 1 characterizes the distributional consequences of the fertilizer price shock in the frictionless economy. Low-α villages see the largest changes in profit when villages are substitutes ($\nu > 1$), with the opposite pattern holding when villages are complements ($\nu < 1$). The intuition relies on understanding how village substitutability affects patterns of equilibrium redistribution. After the shock, equilibrium forces work to redistribute that lost production to other villages. This is why ν matters. If villages are substitutes ($\nu > 1$), the equilibrium tries to source production from the relatively unscathed (in the partial equilibrium sense) low-α villages. On the other hand, with complementarity ($\nu < 1$) this is not viable. The decline in production from high-α villages needs to be replaced with production from those same villages. Thus, the equilibrium shifts the output price changes toward higher-α villages to cushion the blow of the higher input price. Our data will eventually point us toward the $\nu > 1$ case.

These results offer an efficient benchmark for the evolution of equilibrium outcomes after a fertilizer price shock.[21] As we will show, the inability of the frictional

[21] Technically, this is constrained efficient because households cannot trade *ex ante* insurance contracts, but our interest here is on the impact of financial frictions.

economy to replicate the redistribution highlighted in Proposition 1 will be central to the government's ability to affect welfare with a fertilizer subsidy.

4.4 Characterizing the Frictional Equilibrium

We now return to the frictional equilibrium, where each village is subject to $p_x x + wn \leq \phi_j + w$. The baseline economy is characterized by fertilizer state $(p_{x1}^{\text{market}}, \tau_{x1})$, implying farmer price $p_{x1} = (1 - \tau_{x1})p_{x1}^{\text{market}}$. To make the results as sharp as possible, we assume that $\phi_j + w$ is tight enough that it binds for all j at baseline farmer price p_{x1} (which also guarantees it binds at $p_{x2}^{\text{market}} > p_{x1}^{\text{market}}$ for a fixed subsidy level). Thus, we study an economy in which financial frictions are extremely tight, but still heterogenous across villages. This implies that input choices are

$$x_j^c = \frac{\alpha_j(\phi_j + w)}{\gamma p_x} \qquad \text{and} \qquad n_j^c = \frac{\eta_j(\phi_j + w)}{\gamma w} \tag{4.3}$$

where the superscript c is a reminder that these come from the "constrained" frictional equilibrium (and we will sometimes use superscript u to denote the frictionless or "unconstrained" economy).

It will be useful to define the village j deviation of fertilizer (x) and output (y) from its frictionless benchmark,

$$\kappa_j^x := \frac{x_j^u}{x_j^c} \qquad , \qquad \kappa_j^y := \frac{y_j^u}{y_j^c}$$

A high κ_j^x, for example, implies that x_j^c is far below its unconstrained or frictionless level x_j^u. We focus our discussion in terms of the fertilizer but note that κ_j^x and κ_j^y move one-for-one (see footnote 23). Plugging in the relevant input choices yields

$$\kappa_j^x \quad = \quad p_x^{\frac{-\alpha_j}{1-\gamma}} (p_j^u)^{\frac{1}{1-\gamma}} \left(\frac{\gamma \left(\frac{\eta_j}{w}\right)^{\frac{\eta_j}{1-\gamma}} \alpha_j^{\frac{\alpha_j}{1-\gamma}}}{\phi_j + w} \right) \tag{4.4}$$

κ_j^x offers a summary variable we can use to measure how well policy does at moving village j's fertilizer choice closer to its frictionless benchmark, inclusive of price differences between the two equilibria. We start with the following question: if the government does not adjust its subsidy after the market fertilizer price rises from $p_{x2}^{\text{market}} > p_{x1}^{\text{market}}$, what happens to the set $\{\kappa_j^x\}_{j \in \mathcal{J}}$? We will then use this information to show how the optimal fertilizer subsidy should evolve.

Characterizing Production After the Fertilizer Price Shock One approach to answering this question would be to characterize κ^x directly. This unfortunately depends not only on (α_j, ϕ_j) but on the distribution $G(\alpha, \phi)$ and various other parameters that determine equilibrium prices, making characterization difficult or at least a complicated non-linear combination of model features. But our interest is in changes to κ^x, not its level. This offers a way to sidestep many of these issues. Defining $\Delta_t x = x_2 - x_1$ as the difference between the p_{x2} and p_{x1} equilibria, (4.4) becomes

$$\Delta_t \log(\kappa_j^x) \quad = \quad \frac{-\alpha_j}{1-\gamma}\Delta_t \log(p_x) + \frac{1}{1-\gamma}\Delta_t \log(p_{aj}^u) \tag{4.5}$$

Equation (4.5) measures evolution of fertilizer in village j relative to its frictionless benchmark. Applying Lemma 1 to compute village j's frictionless equilibrium price p_{aj}^u then taking a second difference with village k yields a "diff-in-diff" characterization of how the fertilizer gap evolves,

$$\Delta_t \log(\kappa_j^x) - \Delta_t \log(\kappa_k^x) = (\alpha_k - \alpha_j)(\nu - 1)\Delta_t \log(p_x). \tag{4.6}$$

The efficient economy wants to reallocate production toward low-α villages if $\nu > 1$ and high-α villages if $\nu < 1$, which it accomplishes via general equilibrium output price changes. They key insight of (4.6) is that the frictional equilibrium always fails to deliver this redistribution, except in the knife-edge case of Cobb-Douglas aggregation. It keeps too much fertilizer in high-α villages when $\nu > 1$ and too little when $\nu < 1$.[22,23] Together, this lack of productive reallocation manifests in a simple summary of how village-level distortions change, which we summarize in Proposition 2.

Proposition 2. *Assume the market fertilizer price rises from p_{x1}^{market} to p_{x2}^{market} for a fixed subsidy τ_x. Define the shadow value of fertilizer in village j as*

$$1 + \lambda_{jt}^c = \frac{\alpha_j p_{aj} x^{\alpha_j - 1} n^{\eta_j}}{(1-\tau_x)p_{xt}^{market}}.$$

[22] To see this more clearly, rewrite $\kappa_j^x = x_j^u / x_j^c$ as the components of the fraction. For two villages with $\alpha_H > \alpha_L$, writing (4.6) in this manner implies

$$\begin{aligned}
\overbrace{\Delta_t \log(x_H^c) - \Delta_t \log(x_L^c)}^{\equiv\, \Delta \text{ frictional equilibrium}} \quad &> \quad \overbrace{\Delta_t \log(x_H^u) - \Delta_t \log(x_L^u)}^{\equiv\, \Delta \text{ frictionless benchmark}} \quad \text{if } \nu > 1 && (4.7)\\
\Delta_t \log(x_H^u) - \Delta_t \log(x_L^u) \quad &= \quad \Delta_t \log(x_H^c) - \Delta_t \log(x_L^c) \quad \text{if } \nu = 1 && (4.8)\\
\Delta_t \log(x_H^c) - \Delta_t \log(x_L^c) \quad &< \quad \Delta_t \log(x_H^u) - \Delta_t \log(x_L^u) \quad \text{if } \nu < 1. && (4.9)
\end{aligned}$$

The left-hand side measures the relative fertilizer changes between H and L in the frictional equilibrium and compares them to their frictionless counterparts on the right, which were characterized in Proposition 1.

[23] (4.5) and (4.6) link directly to the output because $\Delta_t \log(\kappa_j^x) = (1/\gamma)\Delta_t \log(\kappa_j^y)$. Thus, discussing the results in terms of inputs is equivalent to discussing in terms of output.

The equilibrium fertilizer shadow prices $\{\lambda^c_{jt}\}$ *evolve across villages as*

$$\frac{\partial^2 \Delta_t \log(1+\lambda^c_j)}{\partial \Delta_t \log(p_x^{market})\partial \alpha_j} = \frac{1-\nu}{\nu}$$

for any non-degenerate distribution $G(\alpha, \phi)$*.*

Proposition 2 is the frictional counterpart to the frictionless baseline in Proposition 1, where instead of profit we characterize the Restuccia and Rogerson (2008)-style distortion $1+\lambda^c_j$. Thus, Proposition 2 summarizes how distortions evolve across villages. The equilibrium characterization derived above is reflected here: with substitutability ($\nu > 1$) low-α villages see the largest increase in their distortion despite Proposition 1 highlighting the need to direct production there. The opposite holds with complementarity. In either case, the distortion rises in exactly the villages toward which the efficient economy shifts production.

The economic intuition here revolves around general equilibrium price movements. In the frictionless equilibrium, output price changes incentivize reallocation. But financial frictions dampen the output price elasticity of farmer input choices. Notice, for instance, that the output price p_{aj} does not enter village j's input choices (4.3). The intuition is simple: if j constrained away from profit maximizing input choices at its baseline output price, then the output price rises, there is little that can be done to respond to it. But by eliminating these responses, the reallocation demanded by the frictionless equilibrium is impossible to achieve. This opens scope for a subsidy to deliver it.

Implications for the Optimal Subsidy Proposition 3 takes the preceding characterization and translates into optimal policy. It shows that the subsidy response depends only on how easily the government can utilize other, less exposed villages to replace lost production.

Proposition 3. *Assume a planner chooses a fertilizer subsidy (or tax if negative)* τ_x *to maximize utilitarian welfare,*

$$\max_{\tau_x} \quad \int_j \Big[\zeta \log(c_{aj}) + (1-\zeta)\log(c_{mj})\Big] dj$$

subject to τ_x *being consistent with the resulting competitive equilibrium of the frictional economy, including government budget balance. For any non-degenerate distribution* *of $G(\alpha, \phi)$, the optimal subsidy is such that*

$$\frac{\partial \tau_x^*}{\partial p_x^{market}} \begin{cases} < 0, & \text{if } \nu > 1 \\ = 0, & \text{if } \nu = 1 \\ > 0, & \text{if } \nu < 1 \end{cases}$$

If the marginal distribution of α is degenerate so that all villages have identical α, then $\frac{\partial \tau_x^*}{\partial p_x^{market}} = 0$ for all values of ν.

A seemingly reasonable intuition is that the planner should increase the subsidy to counteract the tighter financial friction, but that fails to account for the characterization above: that she can source production from other, less affected villages. If these less affected villages can feasibly replace that lost production ($\nu > 1$) the planner lowers the subsidy, shifting production shares and lowering the tax burden on all households. Attempting the same policy with complementarity ($\nu < 1$) would imply a large negative impact on the aggregate availability of the agricultural consumption good.[24]

On a technical level, the simplicity of this result follows from the insight that the relevant heterogeneity is captured in the level of the optimal subsidy. That is, while the level of $\tau_x^*(p_x^{\text{market}})$ depends on several features of the model like the joint distribution of technology and financial frictions, the sign of the derivative depends only how production is aggregated. Moreover, note that Proposition 3 relies on the *interaction* of financial frictions and technological heterogeneity. If there are no financial frictions, the subsidy rate naturally falls to zero for any price p_x^{market}. Proposition 3 shows that the subsidy is also constant without technological heterogeneity (but will generically be positive).

While relaxing the assumptions used here eliminates the clarity of the theoretical results, the main message will remain the same: the planner uses its subsidy to induce reallocation across heterogeneous production units that are otherwise limited by financial frictions. Parameters other than just the tightness of financial friction affect both the sign and magnitude of how the subsidy evolves with fertilizer prices. Thus, answering this question both quantitatively *and* qualitatively depends on credibly measuring them. Our goal now is to link these model parameters back to the causal empirical estimates in the full dynamic model, to answer our main question: how much *quantitatively* should the subsidy change in response to the fertilizer price shock.

[24]Proposition 3 does not depend on how the subsidy is financed. In the Appendix, we provide the proof of under alternative financing schemes in which all income is taxed and the case in which only labor market income is taxed.

5 Quantitative Exercise and Model Estimation

5.1 Quantitative Exercise

Our main quantitative exercise works as follows. We start the model with a baseline market fertilizer price p_{x0}^{market} and subsidy level τ_{x0} that all agents assume will remain fixed forever. This generates a baseline stationary equilibrium. We then shock the economy with a deterministic sequence $\{p_{xt}^{\text{market}}, \tau_{xt}\}_{t=1}^{t=\infty}$ that eventually converges back to its baseline values $(p_{x0}^{\text{market}}, \tau_{x0})$ and trace out the transition path of the economy.[25]

Under this assumption, we calibrate the model using the empirical counterparts $\{\widehat{p}_{xt}^{\text{market}}, \widehat{\tau}_{xt}\}_{t=0}^{t=\infty}$ that were realized in Rwanda and restrict ourselves to model outcomes from $t \in \{0, 1, 2, 3, 4, 5\}$ to be consistent with our empirics (the shock occurs at in the first half of 2022 ($t = 1$), and our data runs until June 2024, our $t = 5$). Once calibrated, we will hold fixed the exogenous market price $\{p_{xt}^{\text{market}}\}_{t=0}^{t=\infty}$ and feed in different subsidy paths $\{\tau_{xt}\}_{t=0}^{t=\infty}$ to study counterfactual outcomes. Figure 5 plots what we feed into the model exogenously for calibration.

Figure 5: Model Time Paths for Calibration

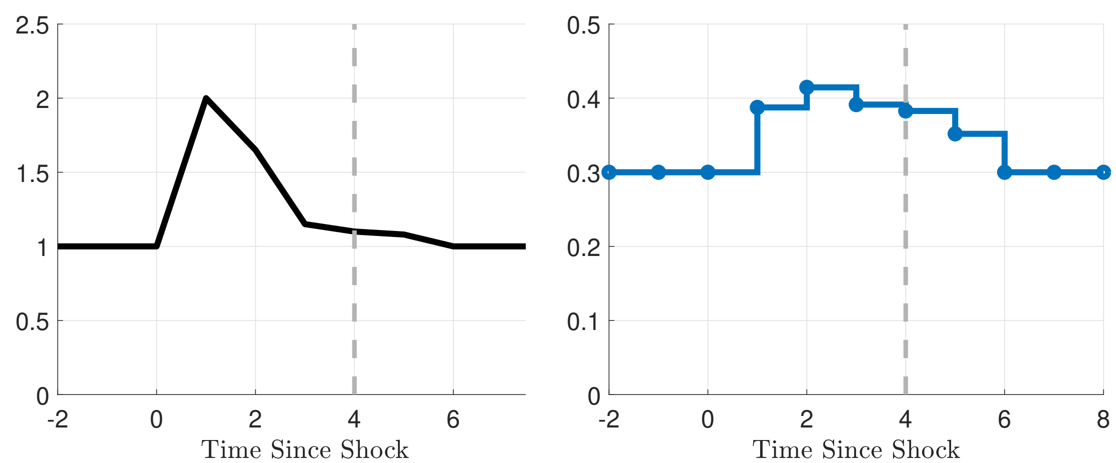

Figure Notes: Panel (a) plots the market price p_x^{market} and Panel (b) the subsidy level τ_x, which is plotted as "constant" within period to match the graphical representation of the earlier empirical results. The gray bar represents when our data collection ends, so the calibration uses only data from the left of that.

[25] The international fertilizer price and Rwandan subsidy level were extremely stable in the decade leading up to 2021 (see Section 2). While this is of course just one realization of a random variable, the expert forecasts in Figure 1 are consistent with beliefs that the price stability would continue. Beliefs consistent with this result would closely approximate our assumption. This comes at the quantitative cost of missing any increased uncertainty on the evolution of fertilizer prices after the shock.

5.2 Estimating Model

The model includes several parameters. On the production side, we require the CES aggregator for agricultural production ν. For village production, we need to estimate the joint distribution of fertilizer intensity α and financial frictions ϕ across villages, $G(\alpha, \phi)$ along with the elasticity between fertilizer and labor σ and returns to scale γ, along with the idiosyncratic shock distribution $F(z)$. In the labor market, we need the Pareto shape parameter for manufacturing skill θ_m. Finally, we require the return to savings R and utility function parameters including the discount rate β and agricultural utility weight α.

5.2.1 Parameters Set Exogenously

We normalize non-agricultural productivity $A = 1$. We also set two parameters outside the model, the discount rate $\beta = 0.95$ and the returns to scale $\gamma = 0.8$.

We also assume that α and ϕ are distributed according to the joint distribution $G(\alpha, \phi)$. We assume that the marginal distribution of α is drawn from a truncated log-normal distribution on (0,1) with parameters $(\mu_\alpha, \sigma_\alpha)$. We assume that the marginal distribution of $1+\phi$ is distributed exponential with parameter λ_ϕ. Finally, we assume they are joined together with a Clayton copula, which gives the cdf

$$G(u_\alpha, v_\phi) = \max\{u_\alpha^\rho + v_\phi^\rho - 1, 0\}^{\frac{-1}{\rho}}$$

where u_α and v_ϕ are the relevant marginal cdfs and ρ controls dependence.[26] This implies 6 parameters to estimate for the joint distribution.

5.3 Using the Empirical Results to Inform Remaining Calibration

We then make use of our empirical results to choose parameters. Given that our empirics are based off shift-share coefficients, we will write X_{j0} as the total fertilizer expenditure at village j at the baseline steady state, so that our exposure measure in village j is $\log(X_{j0})$.

Substitutability Between Village Production, ν The market clearing condition for the final agricultural good producer implies that that village j output price is a function

[26] We choose this particular way to link the two marginals because the Clayton copula has nonzero lower-tail dependence, meaning that we can push our economy toward the lower limit in the (α, ϕ) space without sacrificing variation, while maintaining the simplicity benefits offered by the wider class of Archimedean copulas.

of its production share and the agricultural consumption price,

$$p_{ajt} = \left(\frac{Y_{at}}{y_{ajt}}\right)^{\frac{1}{\nu}} p_{act}.$$

Some algebra implies

$$(1-\nu)\Delta_t \log(p_{ajt}) = \Delta_t \log(p_{ajt} y_{ajt}) - (\Delta_t \log(Y_{at}) + \nu \Delta_t \log(p_{act}))$$

Notice that the last term in parenthesis is constant across villages, which means that

$$(1-\nu)\underset{[+0.030]}{\frac{\partial \Delta_t \log(p_{jt})}{\partial \log(X_{j0})}} = \underset{[-0.041]}{\frac{\partial \Delta_t \log(p_{jt} y_{jt})}{\partial \log(X_{j0})}}. \tag{5.1}$$

The parameter ν depends on the relative movements of village j production to its output price.[27] But more practically, these two terms are explicit formulas for Poisson regression coefficients in a shift-share, as we derived in Section 3. Those values are given in brackets below the equation, and imply $\nu = 2.37$.

Our theoretical results in Section 4 highlight the importance of this parameter. The natural experiment combined with micro data allows us to infer it independently of the remaining model structure. This is our first, and most important, result suggesting that the model will imply a decline in the optimal fertilizer subsidy after the fertilizer price rises.

Production Elasticity σ A similar procedure helps us measure the modern technology production elasticity σ. Conditional on using the modern technology, the cost ratio for household i in village j is

$$\frac{p_{xt} x_{ijt}}{w_{ajt} n_{ijt}} = \frac{\alpha_j}{1-\alpha_j} \left(\frac{w_{ajt}}{p_{xt}}\right)^{\sigma-1}.$$

Denoting S_{jt} as the share of households that use fertilizer in village j at time t, the average cost ratio in village j is

$$\mathbb{E}_i\left[\frac{p_{xt} x_{ijt}}{w_{ajt} n_{ijt}}\right] = S_{jt}\left(\frac{\alpha_j}{1-\alpha_j}\right)\left(\frac{w_{ajt}}{p_{xt}}\right)^{\sigma-1}.$$

[27] A perhaps overly-explicit way to write this equation is

$$(1-\nu)\frac{\partial \Delta_t \log(\mathbb{E}_i[p_{a,ijt}])}{\partial \log(X_{j0})} = \frac{\partial \Delta_t \log(\mathbb{E}_i[p_{a,ijt} y_{ijt}])}{\partial \log(X_{j0})},$$

making use of the fact that prices are identical in the model within each village.

Differencing then taking the derivative with respect to baseline log fertilizer expenditures again gives us the Poisson shift share form:

$$\underset{[-0.057]}{\frac{\partial \Delta_t \log\left(\mathbb{E}_i\left[\frac{p_{xt}x_{ijt}}{w_{ajt}n_{ijt}}\right]\right)}{\partial \log(X_{j0})}} = \underset{[-0.059]}{\frac{\partial \Delta_t \log(S_{jt})}{\partial \log(X_{j0})}} + (\sigma-1)\underset{[+0.068]}{\frac{\partial \Delta_t \log(w_{ajt})}{\partial \log(X_{j0})}}. \tag{5.2}$$

The empirical values are in brackets below their respective terms (the left-hand side cost ratio includes payments to own farm work in the wage bill). Together, they imply $\sigma = 1.03$.

Aside from realism, these results highlight why including the extensive margin matters here. Without it, production function estimates are biased toward complementarity and our empirics would counterfactually point us to $\sigma = 0.16$. As we will show shortly, this parameter has quantitative implications.

Non-Agricultural Labor Ability θ The Pareto distribution assumption of non-agricultural labor efficiency units implies that a household's total non-agricultural earnings e^m_{ijt} are

$$e^m_{ijt} = \left(\frac{\theta}{\theta-1}\right) w_{ajt}^{1-\theta}$$

which implies

$$\underset{[-0.108]}{\frac{\partial \Delta_t \log(\mathbb{E}_i[e^m_{ijt}])}{\partial \log(X_{j0})}} = (1-\theta)\underset{[+0.068]}{\frac{\partial \Delta_t \log(w_{ajt})}{\partial \log(X_{j0})}}. \tag{5.3}$$

Using our Poisson empirical results in the same way as before, we get that $\theta = 2.59$.

Measuring Distortions The theoretical results in Section 4 show that the central feature of the model is interaction between technology differences and financial frictions. Of course, we observe fertilizer use, but not what causes it. Our solution here is to measure model-consistent distortions, so that we are not mis-attributing technological differences to distortions or vice versa.

To operationalize this idea, define λ_{ijt} as the shadow value of in household i in village j at time t, which implies $1 + \lambda_{ijt}$ is the reduced-form distortion faced by the household (in a Restuccia and Rogerson, 2008, sense). For households using fertilizer, the first order conditions imply[28]

$$(1+\lambda_{ijt})^{\sigma} p_{xt} x_{ijt} = \gamma^{\sigma} \alpha_j z_{ijt}^{\frac{\sigma-1}{\gamma}} (p_{ajt} y_{aijt})^{\sigma - \frac{\sigma-1}{\gamma}} p_{ajt}^{\frac{\sigma-1}{\gamma}} p_x^{1-\sigma}. \tag{5.4}$$

This is the same moment whose evolution across villages was formalized in the simpler static model of Section 4 (see Proposition 2). Our goal is to measure it using observables. Taking logs, then averaging over households and villages gives two moments:

$$\Delta_t \mathbb{E}_{ij}[\log(1+\lambda_{ij})] = \left(-\frac{1}{\sigma}\right) \underset{[-0.097]}{\Delta_t \mathbb{E}_{ij}[\log(p_x x_{ij})]} + \left(\frac{\sigma-1}{\sigma\gamma}\right) \underset{[+0.025]}{\Delta_t \mathbb{E}_j[\log(p_{aj})]} \tag{5.5}$$
$$+ \left(1 - \frac{\sigma-1}{\sigma\gamma}\right) \underset{[+0.207]}{\Delta_t \mathbb{E}_{ij}[\log(p_{aj} y_{aij})]} + \left(\frac{1-\sigma}{\sigma}\right) \underset{[+0.67]}{\Delta_t \log(p_x)} + \left(\frac{\sigma-1}{\sigma\gamma}\right) \Delta_t \mathbb{E}_{ij}[\log(z_{ij})]$$

$$\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(1+\lambda_{ij})]}{\partial \log(X_{j0})} = \left(-\frac{1}{\sigma}\right) \underset{[-0.059]}{\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(p_x x_{ij})]}{\partial \log(X_{j0})}} + \left(\frac{\sigma-1}{\sigma\gamma}\right) \underset{[+0.030]}{\frac{\partial \Delta_t \mathbb{E}_j[\log(p_{aj})]}{\partial \log(X_{j0})}} \tag{5.6}$$
$$+ \left(1 - \frac{\sigma-1}{\sigma\gamma}\right) \underset{[-0.086]}{\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(p_{aj} y_{aj})]}{\partial \log(X_{j0})}} + \left(\frac{\sigma-1}{\sigma\gamma}\right) \frac{\partial \Delta_t \mathbb{E}_{ij}[\log(z_{ij})]}{\partial \log(X_{j0})}$$

These equations say something simple despite the cumbersome notation. (5.5) tells us how the average distortion changes after the price shock. (5.6) tells us how that change evolves with baseline fertilizer intensity.

There are two unobservable terms here. The distortion on the left-hand side and the average modern ability on the right-hand side. However, because we estimate $\sigma = 1.03$, the distortion moments are well-approximated by only observable terms. Thus, we can target them directly in our calibration (though note the subtle change in order of operations that implies that the model-consistent regression is OLS instead of Poisson).[29]

Unlike the previous results, measuring distortions in this way does not pin down any parameters directly, because the magnitude of distortions will depend on other

[28] While the CES assumption makes it somewhat less intelligible, note that with $\sigma = 1$ it simplifies to

$$\frac{p_{xt} x_{ijt}}{p_{ajt} y_{aijt}} = \frac{\gamma \alpha_j}{1+\lambda_{ijt}}$$

where $\gamma\alpha_j$ is the exponent on fertilizer in the production function $y = z(x^{\alpha} n^{1-\alpha})^{\gamma}$. (5.4) is just the CES generalization of this familiar Cobb-Douglas result.

[29] In the Appendix we use the Pareto properties of modern productivity to develop a more involved strategy that allows us to use the same intuition in the generic CES case, but it offers little additional quantitative value here given our estimate of σ.

features of the economy like the persistence and distributional assumptions on productivity (see, e.g., Buera and Shin, 2011; Midrigan and Xu, 2014; Moll, 2014). As such, we will use them as moments in the remaining calibration below, allowing us to jointly target the total cross-sectional variation in fertilizer and model-consistent distortions.

5.3.1 Remaining Calibration

There are 11 parameters to set. The interest rate R, the relative productivity of the traditional technology A_T, three parameters governing the modern technology productivity shocks – the minimum value $\underline{z}$, the Pareto shape parameter θ_M, and the probability of updating – and 6 parameters that govern the joint (α, ϕ) distribution, and the utility weight on agricultural consumption ζ

We set the utility weight $\zeta = 0.75$ because the average household spends 75 percent of its expenditures on agriculture (including own production consumed). We match the gross interest rate $R = 1.00$ to the average value of savings to harvest value across households, which is 73 percent.

Our joint distribution of α and ϕ then matches 4 moments. We choose the exponential parameter on the financial frictions and the copula parameter to match the two distortion moments discussed above. We then use the parameters of the marginal α distribution to match the residual variation. Specifically, μ_α matches the average village-level fertilizer share of expenditure harvest value and σ_α matches the log difference between the ninety-fifth and fifth percentiles of the fertilizer expenditure per acre (our source of variation in the shift-share results). This is the sense in which the distortion moments allow us to match the total cross-sectional variation in fertilizer use, while also remaining consistent with the distortions implied by the empirics.

Finally, the last three moments relate to the evolution of modern productivity. We use the minimum support of the productivity distribution $\underline{z}_M$ and the relative productivity of traditional agriculture A_T to jointly match two baseline moments: the baseline average fraction of farmers who use fertilizer and the relative values of harvest yields between those who do and do not use fertilizer.[30] For the latter, we strip out village-level differences when we estimate, so that our moment is the coefficient $\hat{\beta}$ from the regression

$$\log(y_{ij0}) = \gamma_j + \beta \mathbb{1}[\text{use fert}] + \varepsilon_{ij0}$$

Finally, we match the persistence in fertilizer use to the persistence probability ψ

[30] In the Cobb-Douglas limit of modern technology, we have that $\mathbb{E}_j(S_{j0}) \propto \frac{z_M}{A_T}$. Thus, we first get the ratio of the two, then use the yield difference to distinguish them. This second moment is decreasing in the ratio z_M/A_T

with coefficient $\hat{\beta}$ in the lagged regression

$$\mathbb{1}[\text{use fert}]_{ijt} = \gamma_j + \theta_t + \beta \mathbb{1}[\text{use fert}]_{ij,t-1} + \varepsilon_{ijt}$$

We find $\hat{\beta} = 0.41$ which requires a persistence parameter of $\psi = 0.47$.[31]

Table 4 summarizes our full set of parameters. The model matches the chosen moments well.

[31] Interestingly, this low persistence in agriculture productivity has been found in other contexts using either lagged harvest persistence (Donovan, 2021) or production function estimation techniques (Manysheva, 2022), suggesting differences in both steady state costs of misallocation and transition paths relative to sectors with more persistent shocks (Midrigan and Xu, 2014; Moll, 2014).

Table 4: Calibration Summary

Parameter	Description	Parameter Value	Moment	Target Value	Model Value
Set Exogenously:					
β	Discount factor	0.95	1 year period	0.95	0.95
γ	Returns to scale	0.8	Standard value	0.8	0.8
Match Directly to Empirics:					
ν	Agricultural final good aggregator	2.37	Eq. (5.1) in text	2.37	2.37
σ	Modern tech. CES parameter	1.03	Eq. (5.2) in text	1.03	1.03
θ	Shape, non-agr skills	2.59	Eq. (5.3) in text	2.59	2.59
ζ	Utility weight on agricultural consumption	0.75	Avg. agricultural consumption share of expenditures	0.75	0.75
Jointly Chosen to Match Baseline Model Moments:					
μ_α	Mean CES fertilizer parameter	0.03	Avg. village fertilizer expenditure share of harvest value	0.066	0.067
σ_α	Standard deviation of CES fertilizer parameter	1.01	95-5 ratio of village log fertilizer expenditures per acre	3.55	3.55
R	Gross interest rate on savings	1.01	Savings share of income	0.73	0.73
$\underline{z}_M$	Minimum value, modern productivity	0.83	Baseline share of HHs using fertilizer	0.58	0.58
A_T	Relative productivity of traditional agriculture	1.02	Baseline log(yield) gain among fertilizer users	0.80	0.80
Jointly Chosen to Match Time Series Model Moments:					
λ_ϕ	Financial friction parameter	11.3	Eq. (5.5): average Δ distortion	$\approx$ 0.27	0.24
ρ	Copula parameter	0.01	Eq. (5.6): $\partial\Delta$ distortion$/\partial\log(X_{j0})$	$\approx$ -0.027	
θ_M	Shape parameter, modern productivity	4.13	$\partial\Delta_t\mathbb{E}_j\left[\log(S_{jt})\right]/\partial\log(X_{j0})$	-0.059	-0.059
ψ_z	Persistence, modern productivity	0.47	Lagged persistence of fertilizer use	0.41	0.41

Table notes: The $\approx$ symbols in the target values for λ_ϕ and ρ are the Cobb-Douglas approximate values, with the exact method given in the text.

6 Quantitative Results

We begin the quantitative results under utilitarian welfare in Section 6.1 and highlight the key features of the model that generate the optimal time path of the fertilizer subsidy. In Section 6.2, we discuss the welfare implications of the optimal transition from the existing to the optimal subsidy. Finally, Section 6.3 uses these results as motivation to investigate alternative welfare criteria that could help rationalize the data.

6.1 Optimal Policy under Utilitarian Welfare

We start with the optimal path of the fertilizer subsidy under utilitarian welfare, where the planner puts equal weight on all households. The main results are in Figure 6.

Figure 6a compares the optimal subsidy path with that observed in the data. The optimal path starts at 6.5 percent, then falls to 0.3 percent upon the shock. This differs in two ways from what we see in the data. First, the empirical subsidy rises by 50 percent. Second, the baseline level differs substantially.

Figure 6: Optimal Fertilizer Subsidy Time Path

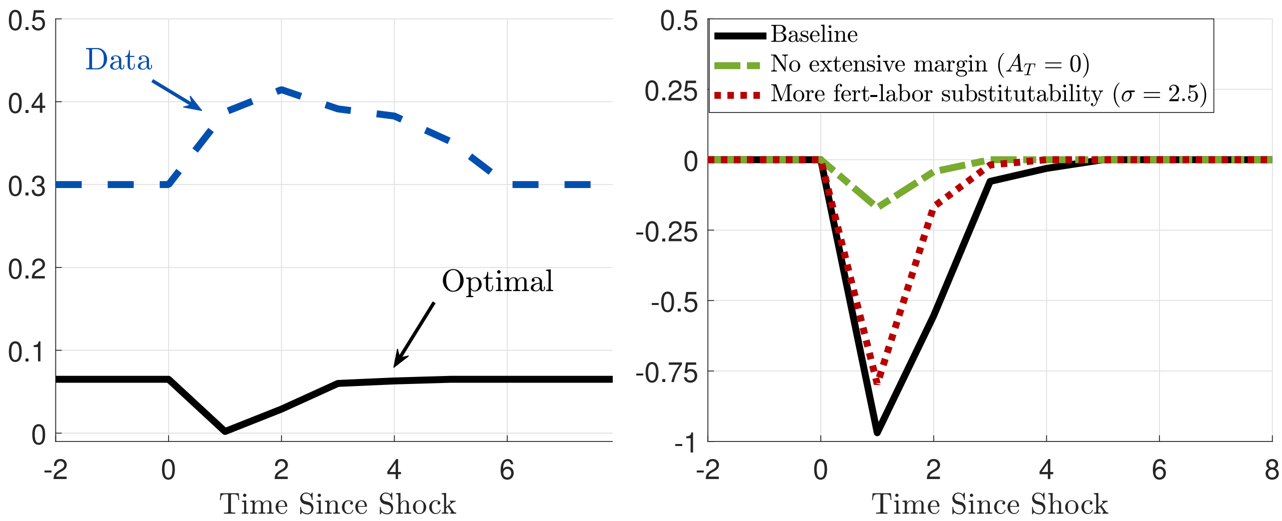

Figure Notes: Panel (a) plots the optimal time path for the fertilizer subsidy, along with the empirically-observed path. Panel (b) overlays two counterfactual time paths for comparison.

Figure 6b plots the optimal subsidy path, along with two counterfactual paths, in percentage changes. That the subsidy falls follows almost entirely from estimating that villages produce substitutes ($\nu > 1$). Central to the quantitative magnitude is farmers' ability to replace fertilizer with labor within their own farm. The planner's goal is to generate more labor-intensive aggregate production, but comes at a distortionary cost of taxing households to pay for the necessary reallocation. Therefore, all else equal, if farmers can easily adjust to relative input price changes, the planner can generate the same aggregate reallocation with a smaller subsidy adjustment. When we raise the modern CES parameter from 1.03 to 2.5, the optimal subsidy becomes flatter; from a 95 percent decline to an 78 percent decline.

On the other hand, the existence of the traditional technology mutes the distortionary impact of lowering the subsidy. The planner can therefore lower the subsidy knowing that farmers in fertilizer-intensive villages can switch to it if the fertilizer price rises too much. This plays an important role here. Turning off the extensive margin by setting traditional productivity $A_T = 0$ implies the subsidy falls by only 17 percent instead of 95.

6.2 Welfare Consequences of Different Policy Decisions

Given the differences between the model-implied optimal subsidy and the observed path in the data, we next use our model to measure the welfare consequences of different policy choices. Table 5 provides the results, with discussion below.

Table 5: Welfare Consequences of Different Policy Decisions

		Consumption equivalent welfare (%)
With Complete Financial Markets		
0.	With $\lambda = \infty$	
Starting from Optimal Steady State $\tau_x^{ss} = 0.065$		
1.	Optimal path	-0.178%
2.	Match %Δ of empirical subsidy	-0.191%
3.	Held fixed at 0.065 after shock	-0.179%
Starting from Empirical $\tau_x^{ss} = 0.30$		
4.	Optimal transition to $\tau_x = 0.065$	0.115%
5.	Exact empirical time path	-0.240%
6.	Held fixed at 0.30 after shock	-0.160%

Table notes: Each measures the welfare cost of full transition path relative to the shock not occurring. This counterfactual stable price equilibrium maintains the listed steady state subsidy rate. With complete financial markets the optimal subsidy is $\tau_{xt} = 0$ for all t.

The first three columns of Table 5 start the economy from its optimal steady state value of $\tau_x^{ss} = 0.065$. Under the optimal subsidy evolution, the price shock manifests in a welfare loss of 0.178 percent, in consumption-equivalent terms. Row 2 attempts to isolate the consequences of the empirically-observed rise in the subsidy on the transition path. To do so, it starts from the optimal steady state value but then requires the subsidy to evolve in the same percentage change observed in the data (see Figure 5). The welfare cost rises to 0.191 percent. Overall, the welfare losses of sub-optimal policy starting from the optimal steady state level are 0.013 percent of consumption equivalent welfare.

The next set of results starts the economy from the observed baseline subsidy of $\tau_x = 0.30$ instead of the optimal 0.065. Imposing the exact path observed in the data implies (again, see Figure 5 for this path) implies a consumption-equivalent welfare loss of 0.240 percent from the shock (Row 5). This subsidy path is costly. Even holding the subsidy fixed at its inefficiently high baseline level offers a 0.08 percent welfare gain compared to the realized path (Row 6)

Finally, Row 4 measures the full consequences of an optimal transition. If the planner can use the unexpected increase in fertilizer prices to optimally transition the economy, the shock becomes beneficial. That is, the average household would be willing to absorb the price shock if it meant transitioning away from the 30 percent subsidy and toward the optimal 6.5 percent. The benefit to the average household is 0.115 percent of consumption-equivalent welfare, relative to continuing under a world of $\tau_x = 0.3$ but with no price shock.

Not only would the average household be willing to make this trade, the average household in each village would be willing to do so. Figure 7 plots the consumption-equivalent welfare gains by village, using its parameter α (the modern technology coefficient on fertilizer) to distinguish villages.

Figure 7: Consumption-Equivalent Welfare by Village Technology (%)

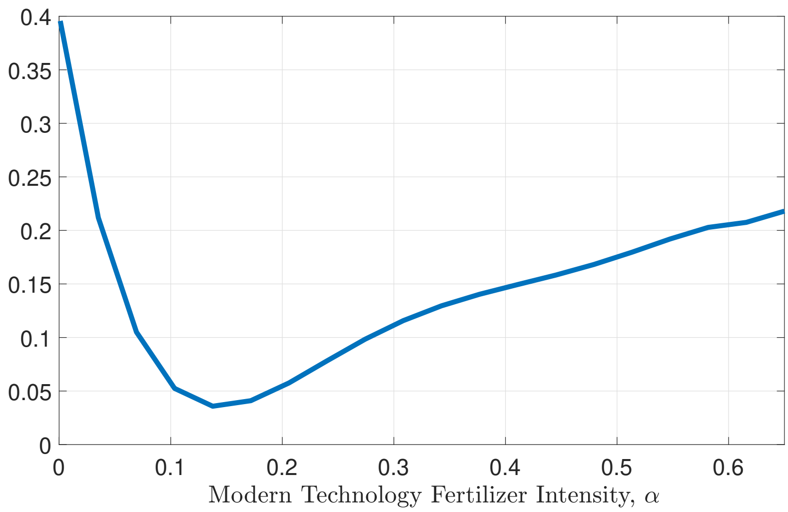

Figure Notes: Consumption-equivalent welfare is measured in percentages, so that 0.4 represents 0.4 percent.

6.3 Rationalizing the Observed Subsidy with Alternative Welfare Functions

If the welfare gains are large enough to compensate households for even the large price shock they faced in 2022, why did this not happen? One possibility is that our utilitarian welfare function is too restrictive. To conclude the paper, we investigate the potential of other welfare criteria to match the data.

We consider three main possibilities: inequality or poverty-reduction preferences, valuing aggregate agricultural production directly (i.e., "food security"), and a direct political economy appeal to stable fertilizer prices for farmers independent of their importance measured via standard welfare. All three are mentioned in planning documents (e.g. MINAGRI, 2018). Our main finding is that the first makes the match between data and model worse. A combination of the second and third can match the data. However, using strategic planing documents produced in pursuit of *ex post* changes to the Rwandan subsidy program, we argue that our required welfare criterion is likely reduced-form representation of a time consistency problem for the government rather than representing the true preferences of the government.

6.3.1 Rawlsian Welfare

Pro-poor redistribution is potentially an important aspect of fertilizer subsidies in the absence of other policy instruments. We compute optimal policy under the limiting Rawlsian case, where the planner values only the minimum across all productivity-savings-village triplets (s, z, j) in the stationary equilibrium.

This cannot rationalize the data for a simple reason: the poor do not use fertilizer. The Rawlsian planner instead wants a 30 percent *tax* on fertilizer in the baseline stationary equilibrium.

Fertilizer use in the baseline equilibrium is not very persistent, a result driven by the frequent shifts on the extensive margin observed in the data. The utilitarian planner internalizes this churn when making policy. She takes seriously that non-fertilizer-users today are likely to use fertilizer in the future. The Rawlsian planner does not: while the name of the poorest household changes period-by-period, households at the bottom of the income distribution use the no-fertilizer traditional technology. The Rawlsian planner therefore prefers to subsidize non-agricultural consumption instead of fertilizer. The same result holds for any intermediate case between utilitarian and Rawlsian with smaller quantitative magnitudes. Overall, fertilizer subsidies are not a pro-poor policy in our model.

6.3.2 Aggregate Food Security Concerns and Smooth Prices

A second view is that fertilizer policy is designed with aggregate food security concerns in mind. To include this, we write down a money-metric welfare criterion that is a convex combination of utilitarian welfare and total agricultural production. It will be helpful for exposition to add in our price stability term as well.

Specifically, define $C^* = e^{(1-\beta)W}$ as the certainty-equivalent consumption that generates utilitarian welfare $W := \mathbb{E}_0 \sum_{t=0}^{\infty} u(c_{at}, c_{mt})$. Our adjusted welfare function, written in money-metric terms, is

$$W_Y := \omega_c P_0^{\text{ref}} C^* + \omega_y \sum_{t=0}^{\infty} \beta^t p_{a0t}^{\text{ref}} Y_{at} + \omega_x \sum_{t=0}^{\infty} \beta^t (p_{xt} - p_{x0})^2, \tag{6.1}$$

where recall that $p_{xt} = (1-\tau_x) p_{xt}^{\text{market}}$. Our reference prices come from the calibration equilibrium in which the subsidy path is that observed in the data, where $P_0^{\text{ref}} = \sum_{t=0}^{\infty} \beta^t P_t$ for price index $P_t = (p_{a0t}^{\text{ref},\zeta}(1+\tau_{mt}^{\text{ref}})^{1-\zeta})/(\zeta^{\zeta}(1-\zeta)^{1-\zeta})$.

These three terms represent utilitarian welfare (with weight ω_c), the value of agricultural production (ω_y), and dislike of fertilizer price variation ($\omega_x < 0$).[32] The baseline utilitarian planner sets $\omega_c = 1$ with $\omega_y = \omega_x = 0$. Section 6.1 shows that this restricted set of welfare weights cannot match the data. We now ask which combinations of $(\omega_c, \omega_y, \omega_x)$ can.

Our strategy works as follows. First we set $\omega_x = 0$ and find the value of ω_y and $\omega_c = 1 - \omega_y$ that match the baseline subsidy level of 30 percent. Since the final term in (6.1) is irrelevant in the stationary equilibrium, we can then set ω_x to match any residual variation in the transition path.

Food Security Alone Matching the baseline subsidy requires $\omega_y = 0.63$ and $\omega_c = 0.37$. Figure 8 plots the implied optimal subsidy under $(\omega_c, \omega_y, \omega_x) = (0.37, 0.63, 0)$, along with the data and our baseline optimal subsidy with $(\omega_c, \omega_y, \omega_x) = (1, 0, 0)$. While the baseline is matched by construction, this welfare criterion still predicts a decline on impact, from 30 to 23 percent peak-to-trough. Thus, allowing the planner to value production directly can match either the baseline level or the increase post-shock, but not both.[33] These results highlight that the re-allocation motive is quantitatively quite powerful here; the planner accepts lower aggregate agricultural production to reallocate where that production occurs.

[32] Note that this last term is a direct valuation of price variation; smooth prices already affect welfare and thus their indirect effect on welfare are captured there.

[33] Production is monotone in the welfare weight ω_y. Thus, an equivalent interpretation of these results is that we can choose ω_y to match the increase after the shock or the baseline, but there is not a single ω_y that matches both.

Figure 8: Optimal Fertilizer Subsidy Time Path

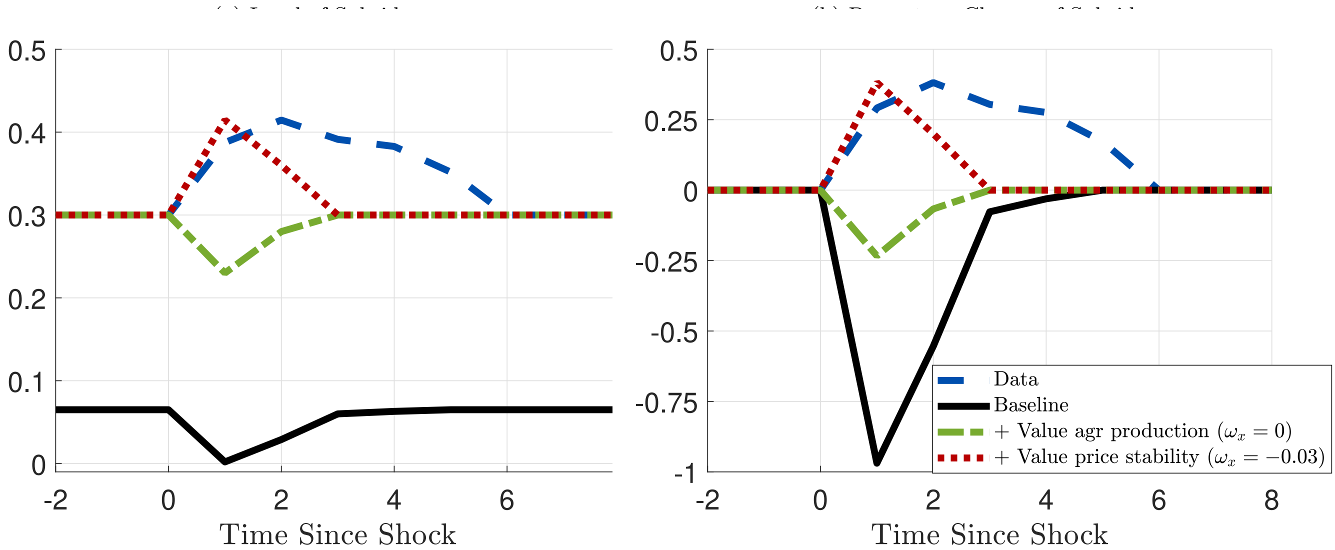

Figure Notes: Panel (a) plots the optimal time path for the fertilizer subsidy under two conditions. First, with the restriction that $\omega_x = 0$, we choose ω_y such that the baseline level matches the empirical baseline subsidy level. Second, we set ω_x such that the maximum of the implied optimal subsidy equals the maximum of the empirical subsidy. We include the baseline optimal subsidy and the data for comparison. Panel (b) plots the same, but in percentage differences.

Adding Smooth Price Concerns Directly Since our last term – that governs the relative importance of smooth prices directly – does not appear in steady state calculations, we can match the time series path with ω_x independently of the previous results. We find that $\omega_x = -0.03$ implies that the optimal subsidy rises by the same amount as the data. $(\omega_c, \omega_y, \omega_x) = (0.37, 0.63, -0.03)$ can match both the level and change on impact. The results are in Figure 8.[34]

Overall, this section shows that issues related poverty-led redistribution are unlikely to account for the divergence between optimal policy and realized policy. Instead, we need to appeal directly to a preference for farmer fertilizer price stability. The natural question then is what these adjustment costs represent. For that, we turn to government policy documents that discuss these issues explicitly.

What does this adjustment cost represent? Our overarching conclusion here is that these adjustment costs are a reduced-form stand-in for time consistency issues that plague industrial policy globally (see Juhász and Lane, 2024, for a review). Our evidence for this view comes from the paper trail left by the government's debate around phasing out fertilizer subsidies, which we discuss here.

Rwanda's fertilizer subsidy program was put in place as a medium-term develop-

[34] The model has a difficult time rationalizing the fact that the subsidy remains high even after the real market price returns to its baseline level.

ment strategy, not as a short-term stabilizer against input price variation. Yet the program was sold to farmers as a way to keep prices low and stable, including several of the changes to the program discussed in Section 3. Together, these changes implied that farmers had come to see the subsidy program as a social support program instead of a longer-term development program (IRDP, 2022; Spielman et al., 2025).

This interpretation was irrelevant in the long period of stable prices: farmer views and government incentives aligned. But the government understood the potential for divergence. In fact, they commissioned a series of consulting reports to study how to roll back the subsidies and replace them with more direct policy prescriptions focused on the underlying market failures: credit, insurance, and (in pursuit of self-financing) savings vehicles (MINECOFIN, 2024; World Bank, 2025). These reports mention the divergent expectations between farmers and the government explicitly. For example, IRDP (2022) notes in a section of farmers' mindset about cost ownership that "the farmers believe that it [price fluctuations] is the government's responsibility. (pg. 30)" These forces conflicted with optimal policy during this unprecedented price increase.

While the evidence above is from Rwanda, the myriad of political complications associated with lowering fertilizer subsidies are well appreciated (Jayne et al., 2018). In this particular case, the same pattern played out in many other developing countries.[35] As such, we view understanding the political details that link longer-term development policy with short-term stabilization as an important issue for future work.

7 Conclusion

How should policy be designed in a setting with financial frictions and heterogeneous technology? These are core features of developing countries. We use a quantitative general equilibrium model, primary data, and a large natural experiment that doubles real fertilizer prices in Rwanda to help answer this question. Our results highlight the importance of technological heterogeneity on changing not just the quantitative magnitude but qualitative sign of subsidies after a shock, and the quantitative magnitudes depend on several parameters that govern aggregate elasticities like the availability of alternative "traditional" technologies and the shapes of production function. Though we use Rwandan agriculture as our focus here, both the policy attention and forces at play in our model are general considerations for policy.

[35] Burundi raised their subsidy from 40 to 60 percent in 2022. Ghana moved from free to only partially subsidized fertilizer for cocoa in 2019, only to resume free fertilizer in 2023. Kenya re-introduced a general fertilizer subsidy previously scrapped in 2020. Tanzania implement an explicitly temporary fertilizer subsidy in 2022. Zambia held farmer prices fixed in 2022, implicitly raising the subsidy level.

In addition to political economy issues discussed in the previous section, how these short-term shocks affect long-run implications is of critical importance. We know, for example, that long run elasticities of substitution for intermediates can be substantially different from short-run elasticities (Peter and Ruane, 2022). In fact, in late 2023 the Rwandan government created a joint venture with a Moroccan fertilizer company to create a fertilizer processing plant in Rwanda, while Nigeria opened its own fertilizer production facility in 2024. The former does not necessarily solve issues related to the procurement of inputs like natural gas. Alternatives may also include new technologies that deliver nitrogen via different means, such as biofertilizers (Schültz et al., 2018). Thus, as short-run stability needs and longer-run technological advances continue, policy will need to evolve along with them.

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Table of Contents for Online Appendix

A Additional Empirical Results

A.1 Natural Gas Prices and Global Fertilizer Prices

Figure 9 plots the relationship between natural gas prices and fertilizer prices since 2014 from the World Bank Commodity Price Outlook (World Bank, 2024), normalized by January 2014. The correlation between the gas prices and the two fertilizer prices are 0.67 and 0.69 for DAP and urea. Figure 9 also shows that two key fertilizer prices more than double in just two years between 2020 and 2022.

Figure 9: Fertilizer Prices and Natural Gas Prices

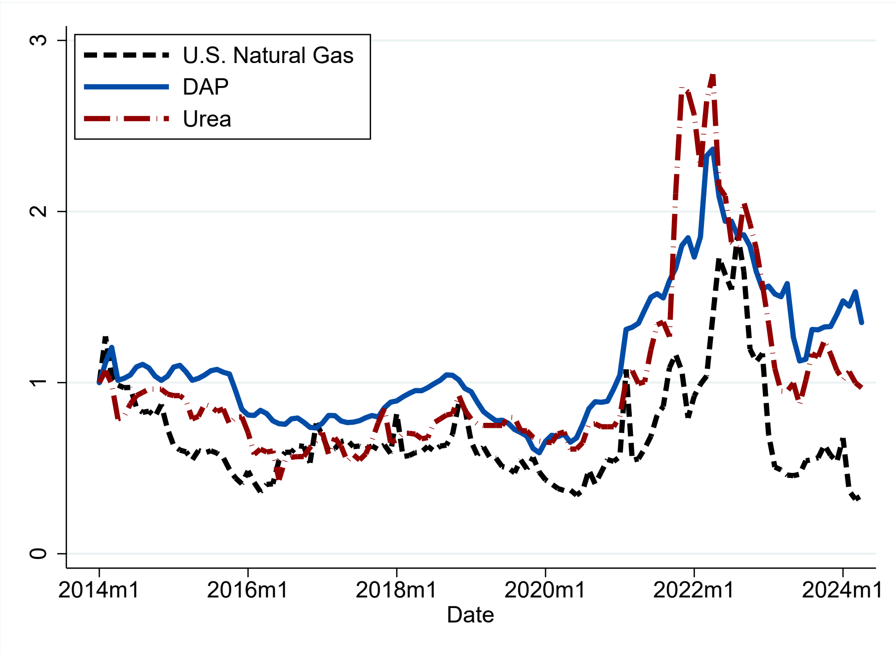


Figure Notes: Nominal monthly prices of DAP, urea, and U.S. natural gas from World Bank (2024). 2014m1 normalized to one.

A.2 Fertilizer versus Other Potential Shocks

Since Russia's invasion of Ukraine affected many potential margins, a natural question is the extent to which the shock we focus on is relevant or co-moves with other substantial shocks. We discuss this issue here.

A.2.1 Additional Data Used Here

In addition to our household level data, in 2021 we began collecting data from traders in the markets associated with these villages. In the 2020 baseline we asked households to tell us about the markets that they bought and sold goods. Any market mentioned by more than 2 households is included in the study, though in practice there are rarely more than one or two markets per village. These data are collected collected monthly by in-person enumeration. This high-frequency collection requires surveys to be relatively short so that enumerators can travel to many markets over the short time horizon. This survey asks questions about goods stocked, inventory, and prices. We make use of that data here to study prices.

A.2.2 Fertilizer Versus Other Intermediate Inputs

The main argument put forth here is that the concentration of fertilizer in a small subset of countries makes it particularly vulnerable to exogenous variation in things like natural gas markets. Other intermediates – pesticide, fungicide – do not share this feature as they can be more easily sourced locally.

Figure 10 plots the raw log per-unit prices for two main intermediate inputs over time derived from the vendor survey. The first is DAP, a key fertilizer in Rwanda as discussed in the text. The second is the main fungicide used, Roket. The 95 confidence intervals are constructed via standard errors clustered at the market level. Each contains a dashed line at January 2022, where prices are normalized to zero. There is a strong increase in DAP prices and none in fungicide prices.

Figure 10: Raw Prices for Inputs (2021-2024)

(a) Fertilizer (DAP) (b) Fungicide (Roket)

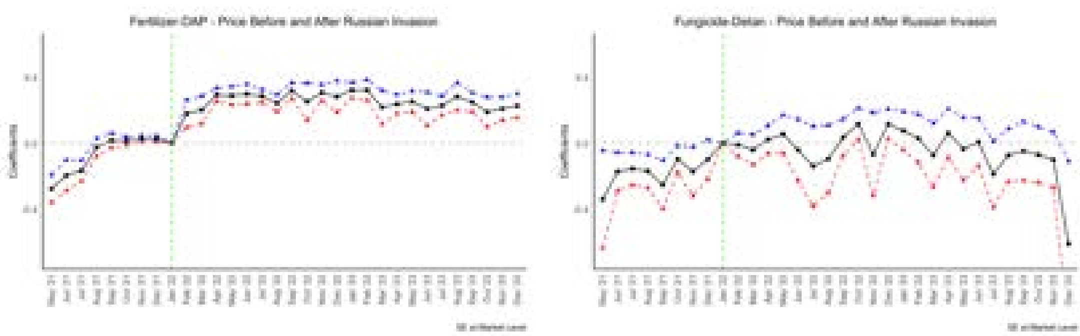


Figure Notes: This figure plots log prices for listed item over time as averages over all vendors. Dashed lines indicate 95 percent confidence interval with standard errors clustered by market.

To study these patterns more systematically we exploit the panel dimension of the data and run a series of regressions

$$\log(p^j_{imt}) = \alpha + \beta Post_t + \theta_i + \eta_m + \nu \log(t) + \varepsilon_{it} \tag{A.1}$$

where, for each product j, we regress the log price on a dummy $Post_t = 1$ if post-January 2022. We control for vendor i's fixed effect θ_i, month effects γ_m for seasonality, and $\log(t)$ is a linear time trend. The results are in Table 6 and confirm the patterns from the raw data in Figure 10.

Table 6: Price Changes Post-January 2022 for Intermediate Inputs

Inputs	Fertilizer			Other Intermediates		
	DAP	NPK	Urea	Roket	Detan	Ridomil
Post-January 2022	0.35***	0.39***	0.33***	0.41	0.14*	-0.18
	(0.03)	(0.05)	(0.08)	(0.26)	(0.07)	(0.24)
Observations	1,658	1,227	1,605	3,172	2,623	1,192
R-squared	0.44	0.43	0.14	0.61	0.27	0.41

Table notes: Standard errors clustered by market are in parentheses. Ridomil is a fungicide and Rocket and Detan are insecticides used in Rwanda. All regressions include trader and month fixed effects and a linear time trend. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

A.2.3 Fertilizer Versus Exogenous Food Supply Shocks via Imports

Another possibility is that the direct import collapse drives agricultural prices higher. Such an argument would be difficult to reconcile with the shift-share results that price changes vary systematically across space with baseline fertilizer intensity. Regardless, we offer additional direct evidence in this section that suggests the changes to agricultural output prices are linked to changes to fertilizer and domestic agricultural productivity changes.

First, we measure direct exposure to Ukrainian imports in Rwanda. All trade data in this section is take from CEPII's BACI database (Gaulier and Zignago, 2010), January 2025 release. On an aggregate level, Ukraine accounts for almost no imports into Rwanda. Ukraine accounted for 0.1 percent of the total value of imports into Rwanda and 0.6 percent of its agricultural imports (HS17 2 digit codes 01-15). Table 7 shows the main products that Rwanda imports from Ukraine, including the share of Ukrainian imports each accounts for.

The only raw agricultural product imported is wheat, which Ukraine exports globally. It makes up 15.5 percent of Rwandan imports from Ukraine and 40 percent of

Table 7: Share of Import Value from Ukraine in 2020

Product Code	Description	Ukranian Import Share
151219	Vegetable oils: sunflower seed or safflower oil	0.303
721391	Iron or non-alloy steel	0.275
110100	Wheat or meslin flour	0.155
680422	Millstones, grindstones, grinding wheels and the like	0.066
220890	Spirits, liqueurs and other spirituous beverages:	0.065
210210	Yeasts: active	0.052
841210	Engines: reaction engines, other than turbo-jets	0.028
880310	Aircraft and spacecraft: propellers and rotors and parts thereof	0.018
902610	Instruments and apparatus: for measuring or checking the flow or level of liquids	0.011

Table notes: This table lists all 6 digit products that make up more than 1 percent of the value of Ukrainian imports into Rwanda in 2020. Product codes are HS17 and a brief description is given in the second column.

total Rwandan wheat imports. Wheat is not produced in any serious quantity in Rwanda. In the 2020 Agricultural Household Survey (NISR, 2020), only 3.5 percent of households grow any wheat. No one in our dataset produces wheat. Thus, exposure to wheat shocks are limited here. This contrasts with other countries like Ethiopia (Adamopoulos and Leibovici, 2024).

Figure 11 offers additional evidence from a different angle. It plots the price patterns for two main staple crops that are consumed domestically and not imported. Despite this, both see their prices rise with a few month lag, suggesting an important equilibrium spillover onto food prices for a domestic productivity shock.

Figure 11: Raw Prices for Local Crops (2021-2024)

(a) Beans (b) Sweet Potatoes

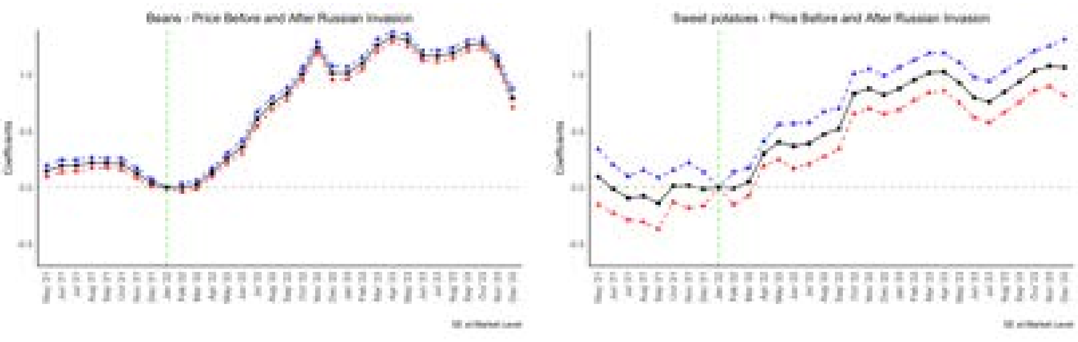


Figure Notes: This figure plots log prices for listed item over time as averages over all vendors. Dashed lines indicate 95 percent confidence interval with standard errors clustered by market.

A.3 Comparing Our Data to Nationally-Representative Data

About every 3 years, Rwandan conducts their Agricultural Household Survey (AHS), which is representative household data that collects information on agricultural practices. We compare our baseline data in 2020 to the 2020 AHS (NISR, 2020). Figure

12 plots the distribution of hectares cropped across households in the AHS and our dataset; the distributions are quite similar. We also compare our sample on the likelihood of using inorganic fertilizer in panel (b) (the AHS reports only an indicator for inorganic fertilizer use, not quantity or expenditure). Our sample somewhat over-represents fertilizer use relative to the national average, though it matches the log-linear shape with land size quite well.

Figure 12: Comparison to Nationally Representative Data in 2020

(a) Farm Size Distribution

(b) Likelihood of Using Inorganic Fertilizer

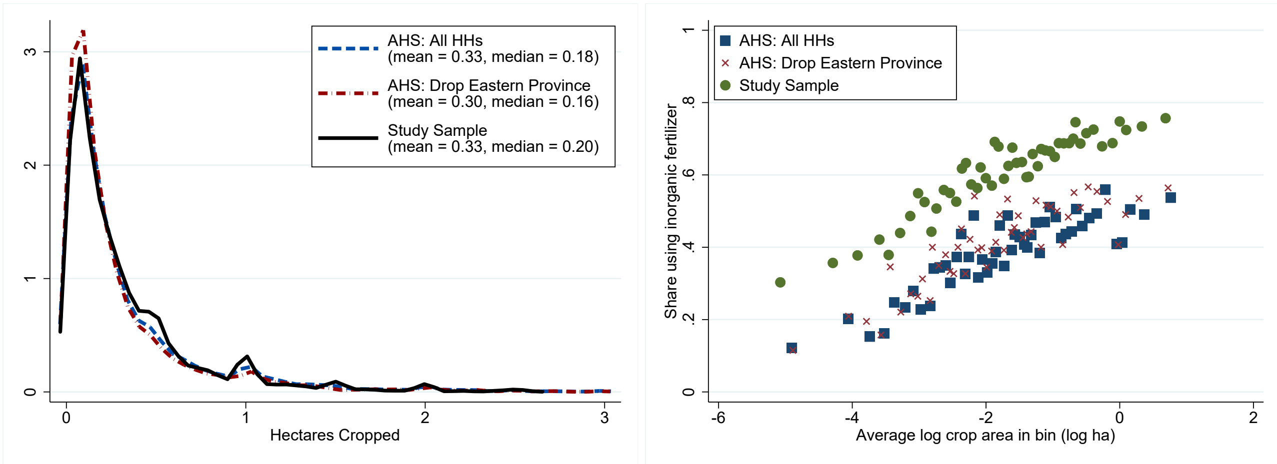


Figure Notes: This figure compares the moments in our baseline study sample in 2020 to nationally representative data from the 2020 Rwandan Agricultural Household Survey (AHS). The top 1 percent of observations are trimmed for farm size. For panel (b), we bin farm size into 30 equally spaced bins and report the averages.

A.4 Market-Level Interactions

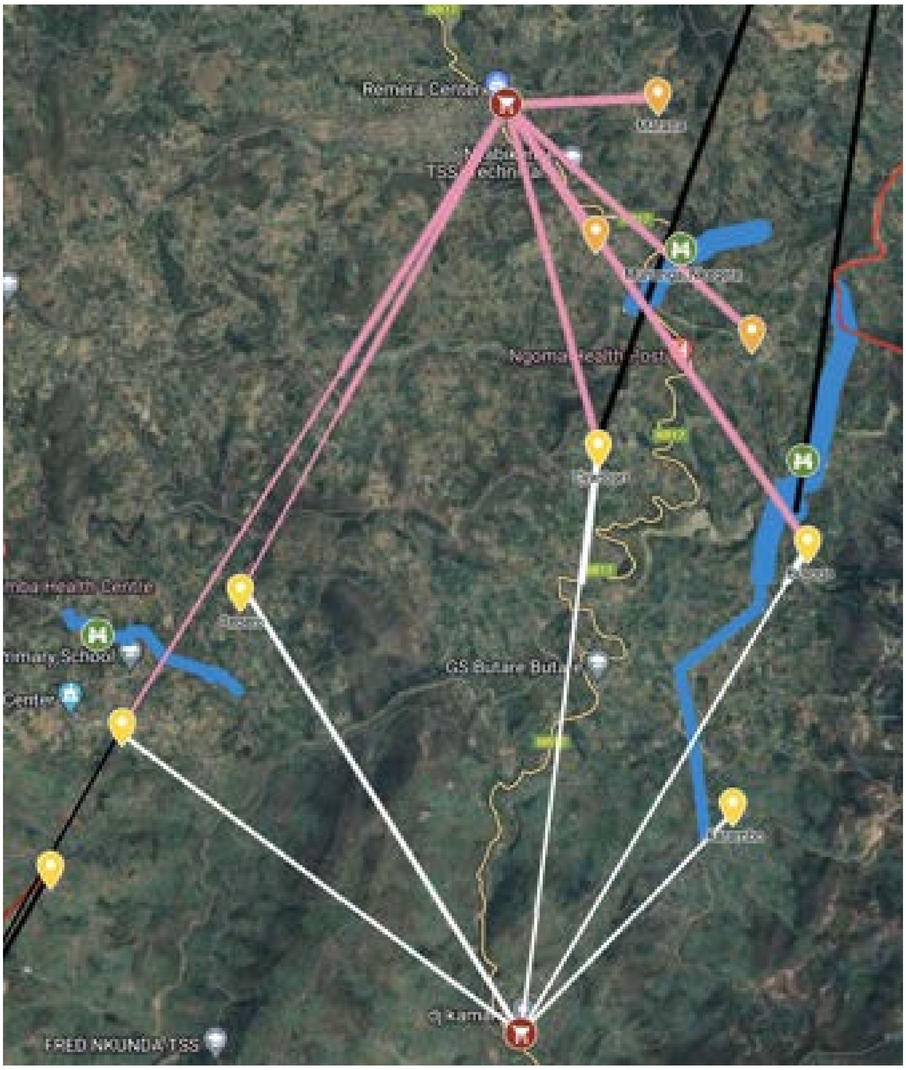
Table 8: Crop Prices

VARIABLES	All (1)	All (2)	Low Market (3)	High Market (4)
Post	0.023***	-0.074***	-0.078***	-0.061***
	(0.009)	(0.014)	(0.020)	(0.023)
Post x Village Fertilizer Intensity		0.032***	0.032***	0.012
		(0.006)	(0.009)	(0.010)
Observations	59,150	59,145	30,524	28,621

Table notes: Standard errors clustered by village are in parentheses. All outcome variables are in logs and regressions are run as OLS. Statistical significance at 0.10, 0.05, and 0.01 is denoted by *, **, and, ***.

Villages do not each sell to unique markets. Figure 13 shows 9 villages (the yellow pins) that interact in two different markets (the red shopping carts). We would only expect the market price to respond to a village's own fertilizer intensity if it made

Figure 13: Example of Village Links to Markets



up an important share of the market-level fertilizer intensity. As a robustness check, we therefore measure the residual fertilizer intensity of the market by computing the average fertilizer intensity of market-connected villages, given by (A.2), and test how prices adjust.

$$\tilde{f}_{Mv0} = \log\left(\frac{ \overbrace{\sum_{\hat{v}\in M_v}\sum_{h\in\hat{v}} p_{x0} f_{h\hat{v}0}}^{\text{fert. for hhs that trade in same market}} - \overbrace{\sum_{h\in v} p_{x0} f_{hv0}}^{\text{own fert.}} }{ \underbrace{\sum_{\hat{v}\in M_v}\sum_{h\in\hat{v}} \ell_{h\hat{v}0}}_{\text{land for hhs that trade in same market}} - \underbrace{\sum_{h\in v} \ell_{hv0}}_{\text{own land}} } \right) / |M_v| \tag{A.2}$$

(A.2) takes total baseline fertilizer for all households that trade in the same market as village v net of that fertilizer from v, and similarly for land, to compute a net fertilizer intensity for the market. If $\tilde{f}_{Mv0}$ is high, then village v makes up only a small portion of the market's fertilizer intensity. We would not expect output prices to respond to village v's fertilizer intensity in this case. On the other hand, if $\tilde{f}_{Mv0}$ is low, village v plays an important role in market M.

Columns (3) and (4) replicate the same regression as Column (2) of Table 2, but breaks the sample into those with high market-connected fertilizer intensity and those with low market-connected fertilizer intensity. The output price in v responds to v's fertilizer intensity only when v is a sufficiently large share of the market.

B Competitive Equilibrium of Simplified Model

Given a subsidy level τ_x and market price of fertilizer p_x^{market}, a competitive equilibrium of this model is a set of decision rules for households $(c_{aj}, c_{mj}, x_j, n_{aj})$, the manufacturing firm N_m, and the final agricultural good producer y_{aj}, prices $\{p_{aj}\}, p_{ac}, w_a$ and tax rate τ_m such that (i) the household's decision rules are consistent with its optimization problem given prices and taxes, (ii) N_m solves the manufacturing firm's decision problem, (iii), y_{aj} solves the agricultural final goods firm decision problem, and (iv) the government balances its budget and markets clear:

1. Government budget balance: $\tau_x p_x^{market} \int_j x_j dj = \tau_m \int_j c_{mj} dj$.

2. Market clearing:

   (a) Agricultural intermediate goods market: for each j, $y_{aj} = x_j^{\alpha_j} n_j^{\eta_j}$

   (b) Agricultural final goods market: $\left(\int_j y_j^{\frac{\nu-1}{\nu}} dj\right)^{\frac{\nu}{\nu-1}} = \int_j c_{aj} dj$

   (c) The manufacturing labor market: $N_m = 1 - \int_j n_j dj$, which amounts to the condition that $w_a = w_m = A$.

C Additional Quantitative Results

C.1 General CES Procedure for Estimating Model-Consistent Distortions

These equations have two important features. First, the right-hand side terms are *almost* all directly observable, except for the change in productivity of modern farmers, a complication induced by technology choice in non-Cobb-Douglas settings (though our estimate of $\sigma = 1.03$ implies almost no weight on this term). Second, there is a subtle difference in the ordering of operations. Here we take logs first before averaging, so the model-consistent regressions are OLS instead of Poisson. Those coefficients are reported under the relevant terms.

Our goal is to use (5.5) and (5.6) as targets in calibration so that we can directly target distortions. $\sigma = 1.03$ implies that the main source of variation comes from the observable terms related to fertilizer expenditures and harvest values, as expected in a Cobb-Douglas setting. Under exactly Cobb-Douglas, we have $\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(1+\lambda_{ij})]}{\partial \log(X_{j0})} = -0.027$. But we do not need to resort to an approximation. We can overcome the un-observability of the final selection term by appealing to the Pareto properties of modern ability, which we summarize in Proposition 4.[36]

Proposition 4. *The expected log productivity of modern farmers is related to the expected share of households that use the modern technology S_{jt} by the relationship*

$$\Delta_t \mathbb{E}_{ij}[\log(z_{ij})|\textit{use modern tech}] = \left(\frac{-1}{\theta_M}\right) \Delta_t \mathbb{E}_j[\log(S_j)], \tag{C.1}$$

and therefore

$$\frac{\partial \Delta_t \mathbb{E}_{ij}[\log(z_{ij})|\textit{use modern tech}]}{\partial \log(X_{j0})} = \left(\frac{-1}{\theta_M}\right) \underset{[-0.059]}{\frac{\partial \Delta_t \mathbb{E}_j[\log(S_j)]}{\partial \log(X_{j0})}} \tag{C.2}$$

This result, which fully accounts for how borrowing constraints evolve with the shock, offers us a way to target distortions directly for any given modern productivity shape parameter θ_M because we can observe changes to the extensive margin of fertilizer use.

Intuitively then, our full strategy is as follows. First, we pick θ_M. We then simulate

[36] Practically, assuming Cobb-Douglas directly saves little time in the estimation procedure. Doing so guarantees only that the distortion moments do not vary with our parameter vector guesses. It does not change the number of parameters to calibrate. Regardless, (5.5) and (5.6) vary little with changes to parameter guesses given $\sigma = 1.03$.

our model until we match 10 moments, 2 of which are the distortion measures in (5.5) and (5.6) implied by the selected θ_M. Finally, we check whether or not the implied model is consistent with the empirical variation in modern technology use across villages, $\partial \Delta_t \mathbb{E}_j \left[\log(S_{jt})\right] / \partial \log(X_{j0}) = -0.059$, implying that our parameters are internally consistent with the variation used to measure distortions.

C.2 Including the Value of Agricultural Production Directly in the Welfare Function

Another rationale for subsidizing fertilizer is targeting directly agricultural production levels in pursuit of goals like domestic food security. Combining this with some measure of household welfare requires requires them to be in the same units. Therefore, we restate the government's welfare function with a money-metric criterion, which requires a bit of notation.

Defining Money-Metric Welfare We define $\widetilde{W}(\{\tau_{xt}, p_{xt}^{\text{market}}\}; \mu_0)$ as the utilitarian value of entering time 0 with a future path of subsidies and market prices $\{\tau_{xt}, p_{xt}^{\text{market}}\}_{t=0}^{\infty}$ and initial distribution $\mu_0(s, z, j)$. This implies a constant level of expenditures $e^*(\{\tau_{xt}, p_{xt}^{\text{market}}\}) \in \mathbb{R}_{++}$ that makes the average household indifferent

$$\frac{\log(\alpha^\alpha (1-\alpha)^{1-\alpha}) - \alpha \log(p_{ac0}) - (1-\alpha)\log(1+\tau_{m0}) + \log(e^*)}{1-\beta} = \widetilde{W}(\{\tau_{xt}, p_{xt}^{market}\}; \mu^0).$$

for reference prices p_{ac0} and τ_{m0}. The value e^* is now a resource-denominated value of welfare. However, it is not independent of the baseline prices chosen, and we therefore choose our reference economy as the baseline stationary equilibrium, with subsidy $\tau_x = 0.3$ and associated stationary equilibrium prices $(p_{ac}^{ss}, p_{aj}^{ss}, \tau_m^{ss})$.

Finding the Welfare Weight to Rationalize the Baseline Subsidy Notice then that if we focus on constant p_x^{market} and τ_x, we can compute the welfare-equivalent expenditures for any stationary equilibrium by setting $\mu^0 = \mu^{ss}(\tau_x, p_x^{\text{market}})$ (we will drop the dependence on p_x^{market} since it does not endogenous here). Specifically, the expenditure level $e^*(\tau_x) - e^*(\tau_{x0})$ is the compensating variation required to make the average household equally well off in the stationary equilibrium τ_{x0} reference economy as in the τ_x new one.

This puts welfare in resource terms and allows us to generalize the government's welfare criterion as

$$\max_{\tau_x} \ \omega e^*(\tau_x) + (1-\omega) p_{a0} Y_a(\tau_x).$$

ω controls the weight on welfare compared to the value of agricultural production, with $\omega = 1$ returning utilitarian welfare. Our goal is to find the value ω such that the optimal subsidy is consistent with the empirical one of $\tau_x = 0.3$ observed in 2020. This occurs when $\omega = 0.57$.

Optimal Subsidy Path Under New Welfare Criterion We then recompute the optimal subsidy path under this new welfare criterion,

$$\max_{\{\tau_{xt}\}} \frac{\omega}{1-\beta} e^*(\{\tau_{xt}\}) + (1-\omega) p_{ac0} \sum_{t=0}^{\infty} \beta^t Y_{at}(\{\tau_{xt}\})$$

where we discount the stream of agricultural production with the household's discount factor and use $\omega = 0.57$ that rationalizes the observed subsidy in 2020. Again, the reference prices here are those from the stationary equilibrium with $\tau_x = 0.3$.

Here, we find that the optimal subsidy still falls on impact, but from 30 to only 29 percent. Thus, valuing agricultural production dampens the incentives to lower the subsidy rate but does not eliminate it.

By virtue of the fact that production value increases monotonically in the subsidy, there is always a welfare weight that rationalizes the observed percentage change in the subsidy. By the same logic, there is always one that rationalizes the baseline value. These results show that the two do not coincide: matching the change requires over-estimating the baseline subsidy, or matching the baseline requires under-estimating the change.

As a more general quantitative point, the inter-village redistribution channel at the heart of our model seems to be quite quantitatively powerful. Even allowing the planner to value production directly does not eliminate the incentives to reallocate production toward more labor intensive villages. While this technological heterogeneity has a natural rationale in agriculture via location-specific characteristics like soil, models focused on the same financial frictions used here tend to do so in the context of identical technologies (Buera et al., 2015a; Itskhoki and Moll, 2019; VanVuren, 2025). Expanding these models to consider policy in the presence of location-specific issues more central to spatial models seems an important avenue for future work.

D Proofs

D.1 Characterization of Frictionless Equilibrium and Proof of Lemma 1

Proof. The proof of Lemma 1 proceeds essentially by guess-and-verify. Specifically, we hypothesize that there exists some value $\mathcal{Y}$ such that the following results hold:

$$\frac{p_{ac}^2}{p_{ac}^1} = \frac{Y_a^1}{Y_a^2} = \mathcal{Y}$$

We then show that there is only one value for $\mathcal{Y}$ that satisfies the market clearing conditions and optimality conditions.

Constructing the Candidate $\mathcal{Y}$ From market clearing for village production

$$p_{aj} = \left(\frac{Y_a}{y_{aj}}\right)^{\frac{1}{\nu}} p_{ac}$$

Therefore,

$$\frac{p_{aj}^2}{p_{aj}^1} = \left(\frac{Y_a^2}{Y_a^1}\right)^{\frac{1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}} \left(\frac{p_{ac}^2}{p_{ac}^1}\right) = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}}. \tag{D.1}$$

Using this price ratio in combination with the village farm first order conditions implies

$$\frac{y_{aj}^2}{y_{aj}^1} = \left(\frac{p_{aj}^2}{p_{aj}^1}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j}{1-\gamma}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu(1-\gamma)}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-\gamma}{\nu(1-\gamma)}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j}{1-\gamma}}$$

which, re-arranging, gives

$$\frac{y_{aj}^2}{y_{aj}^1} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j\nu}{\nu-(\nu-1)\gamma}}. \tag{D.2}$$

and plugging this result into (D.1) gives the village price ratio of

$$\begin{aligned} \frac{p_{aj}^2}{p_{aj}^1} &= \mathcal{Y}^{\frac{\nu-1}{\nu}} \mathcal{Y}^{\frac{-(\nu-1)\gamma}{\nu(\nu-(\nu-1)\gamma)}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}} \\ &= \mathcal{Y}^{\frac{\nu-(\nu-1)\gamma-1}{\nu-(\nu-1)\gamma}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{\alpha_j}{\nu-(\nu-1)\gamma}} \end{aligned} \tag{D.3}$$

Aggregate to get total production

$$Y_2 = \left(\int y_{2j}^{\frac{\nu-1}{\nu}}\, dj\right)^{\frac{\nu}{\nu-1}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}\, dj\right)^{\frac{\nu}{\nu-1}}.$$

Under the maintained hypothesis that $Y_{a2} = Y_{a1}\mathcal{Y}^{-1}$, then

$$\mathcal{Y}^{-1}\left(\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\, dj\right)^{\frac{\nu}{\nu-1}} = \mathcal{Y}^{\frac{(\nu-1)\gamma}{\nu-(\nu-1)\gamma}} \left(\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}\, dj\right)^{\frac{\nu}{\nu-1}}$$

which can be re-arranged to solve for the candidate $\mathcal{Y}$

$$\mathcal{Y} = \left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\, dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}\, dj}\right)^{\frac{\nu-(\nu-1)\gamma}{\nu-1}}. \tag{D.4}$$

The construction of $\mathcal{Y}$ in (D.4) uses two conditions. First, it uses the relevant farm first order conditions. Thus, it already satisfies the optimality conditions of the village farms. Second, it uses the market clearing condition from the stand-in firm that aggregates village production into the agricultural final good. Thus, it satisfies this market clearing condition by construction. The only remaining non-trivial market clearing condition is that of the demand and supply for the final agricultural good.

Confirming that Agricultural Market Clearing Condition is Satisfied Combining the characterizations of village output quantity and price from equations (D.2) and (D.3) gives farm revenue

$$\frac{p_{a2j}y_{a2j}}{p_{a1j}y_{a1j}} = \mathcal{Y}^{\frac{\nu-1}{\nu-(\nu-1)\gamma}} \left(\frac{p_x^2}{p_x^1}\right)^{\frac{\alpha_j(1-\nu)}{\nu-(\nu-1)\gamma}}$$

and therefore farm profit can be written as (note that this equation is used in the main text)

$$
\begin{aligned}
\pi_{2j} &= (1-\gamma)p_{a2}y_{2j} \\
&= (1-\gamma)\mathcal{Y}^{\frac{\nu-1}{\nu-(\nu-1)\gamma}}\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} p_{a1}y_{1j} \\
&= \underbrace{\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\,dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}\,dj}\right)\left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}}}_{\equiv \text{ adjustment to } \pi_{1j}} \underbrace{(1-\gamma)p_{a1}y_{1j}}_{\equiv \pi_{1j}}
\end{aligned}
$$

Using the fact that $p_{aj} = (Y_a/y_{aj})^{\frac{1}{\nu}}p_{ac}$, total baseline income is

$$
\begin{aligned}
\int_j \pi_{j1}dj + w &= (1-\gamma)\int_j p_{a1j}y_{a1j}dj + w = (1-\gamma)Y_{a1}^{\frac{1}{\nu}}p_{ac1}\int y_{a1j}^{\frac{\nu-1}{\nu}}\,dj + w \\
&= (1-\gamma)p_{ac1}\Big(\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\Big)^{\frac{\nu}{\nu-1}} + w = (1-\gamma)p_{a1c}Y_{a1} + w
\end{aligned}
$$

which remains constant after the price shock

$$
\begin{aligned}
\int \pi_{j2}dj + w &= (1-\gamma)\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\,dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}\,dj}\right)\left(\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} p_{a1j}y_{1j}dj\right) + w \\
&= (1-\gamma)\left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\,dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{aj1}^{\frac{\nu-1}{\nu}}\,dj}\right)\left(Y_{a1}^{\frac{1}{\nu}}p_{ac1}\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu-(\nu-1)\gamma}} y_{1j}^{\frac{\nu-1}{\nu}}\,dj\right) + w \\
&= (1-\gamma)\left(\int_j y_{a1j}^{\frac{\nu-1}{\nu}}\,dj\right)Y_{a1}^{\frac{1}{\nu}}p_{ac1} + w \\
&= (1-\gamma)p_{a1c}Y_{a1} + w.
\end{aligned}
$$

Total agricultural demand is therefore

$$
D_{a2} = \frac{\zeta}{p_{ac2}}\left(\int_j \pi_{j2}dj + w\right) = \frac{\zeta}{\mathcal{Y}p_{a1c}}\left(\int_j \pi_{j1}dj + w\right) = \frac{D_{a1}}{\mathcal{Y}}.
$$

Since $Y_{a2} = Y_{a1}/\mathcal{Y}$, this implies $Y_{a2} = D_{a2}$ as required. ■

D.2 Proof of Proposition 1

Proof. This result is proved in the text. ■

D.3 Proof of Proposition 2

Proof. The first part of the proof requires characterizing the new equilibrium as a function of the baseline equilibrium, and we therefore proceed in a nearly identical manner to Lemma 1. Specifically, define

$$\mathcal{Y}^c = \left(\frac{\int_j y_{a1j}^{\frac{\nu-1}{\nu}} \, dj}{\int_j \left(\frac{p_x^2}{p_x^1}\right)^{\frac{-\alpha_j(\nu-1)}{\nu}} y_{aj1}^{\frac{\nu-1}{\nu}} \, dj} \right)^{\frac{\nu}{\nu-1}}.$$

Following an identical methodology to the proof of Lemma 1, this term characterizes the new equilibrium as a function of the baseline one. Specifically important for this proof is that

$$\frac{p_{aj}^2}{p_{aj}^1} = \left(\frac{Y_a^2}{Y_a^1}\right)^{\frac{1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}} \left(\frac{p_{ac}^2}{p_{ac}^1}\right) = \mathcal{Y}^{\frac{-1}{\nu}} \mathcal{Y} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}} = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{y_{aj}^2}{y_{aj}^1}\right)^{\frac{-1}{\nu}}$$

Plugging in

$$y_{aj}^c = \frac{z_j(\phi_j + w)^\gamma}{\gamma^\gamma} \left(\frac{\alpha_j}{p_x}\right)^{\alpha_j} \left(\frac{\eta_j}{w}\right)^{\eta_j}$$

gives

$$\frac{p_{aj}^2}{p_{aj}^1} = \mathcal{Y}^{\frac{\nu-1}{\nu}} \left(\frac{p_{x2}}{p_{x1}}\right)^{\frac{\alpha_j}{\nu}}.$$

The village j shadow price of fertilizer at time t is

$$1 + \lambda_{jt} = \frac{\alpha_j p_{aj} z_j x^{\alpha_j - 1} n_a^{\eta_j}}{p_{xt}}.$$

Using the relevant input choices and the prices,

$$\Delta_t \log(1 + \lambda_j) = \Delta_t \log(p_{aj}) - \alpha_j \Delta_t \log(p_x) = \frac{\nu - 1}{\nu} \log(\mathcal{Y}) + \frac{\alpha_j(1-\nu)}{\nu} \Delta_t \log(p_x).$$

Taking two derivatives gives the result

$$\frac{\partial^2 \Delta_t \log(1 + \lambda_j)}{\partial \Delta_t \log(p_x) \partial \alpha_j} = \frac{1-\nu}{\nu}.$$

$\blacksquare$

D.4 Proof of Proposition 3

D.4.1 An Additional Lemma

Before proving this result, it is helpful to characterize a few results from the frictional equilibrium. We do those in Lemma 2.

Lemma 2. *Two results are true in any equilibrium of the frictional economy.*

1. *Total economy-wide expenditures $E := \int_j \pi_j dj + A$ do not depend on the market price p_x^{market} or the subsidy τ_x.*
2. *The budget-balancing manufacturing tax τ_m does not depend on market price p_x^{market}.*

Proof.

Claim 1 Using the result that $p_{aj} = (Y_a/y_j)^{\frac{1}{\nu}} p_{ac}$, total expenditures are

$$\begin{aligned}\int_j \pi_{j1} dj &= \int_j p_{a1j} y_{a1j} dj - \mathbb{E}[\phi] = Y_{a1}^{\frac{1}{\nu}} p_{ac1} \int y_{a1j}^{\frac{\nu-1}{\nu}} dj - \mathbb{E}[\phi] \\ &= p_{ac1} \Big(\int_j y_{a1j}^{\frac{\nu-1}{\nu}} \Big)^{\frac{\nu}{\nu-1}} - \mathbb{E}[\phi] = p_{a1c} Y_{a1} - \mathbb{E}[\phi]\end{aligned}$$

But for any value $p_{x2} = p_{x2}^{\text{market}}(1-\tau_{x2})$, we have $p_{c2}/p_{c1} = Y_{a1}/Y_{a2}$ or $p_{c2}Y_{a2} = p_{c1}Y_{a1}$ as derived in the proof of Proposition 2. Therefore, $\int_j \pi_{j2} dj = p_{1c}Y_{a1} - \mathbb{E}[\phi] = \int_j \pi_{1j} dj$ as required.

Claim 2 The government's budget constraint is

$$\tau_x p_x^{\text{market}} \int_j x_j dj = \tau_m \int_j c_{mj} dj.$$

Optimality conditions imply

$$\begin{aligned}x_j &= \frac{\alpha_j(\phi_j + A)}{\gamma p_x^{\text{market}}(1-\tau_x)} \\ c_{mj} &= \frac{(1-\zeta)\pi_j}{1+\tau_m}\end{aligned}$$

where π_j is total expenditures in village j. This implies that the government budget constraint is

$$\left(\frac{\tau_x}{1-\tau_x}\right)\frac{1}{\gamma}\Big(\int_j \alpha_j(\phi_j + A)dj\Big) = \frac{\tau_m}{1+\tau_m}(1-\zeta)E$$

where $E = \int_j \pi_j dj$ is economy-wide expenditures. By the first result that E does not depend on p_x^{market}, this proves the second claim. ■

D.4.2 Proof of Proposition 3

Proof. Denoting π_j as the expenditures of village j, we can write the consumption decisions $p_{ac}c_a = \zeta\pi_j$ and $(1+\tau_m)c_m = (1-\zeta)\pi_j$ which gives the indirect utililty function for village j,

$$\Omega - \zeta\log(p_{ac}) + (\zeta - 1)\log(1+\tau_m) + \log(\pi_j)$$

with constant Ω. Manipulating the agricultural consumption market clearing condition implies

$$p_{ac} = \frac{\zeta E}{Y_a}$$

where $E := \int_j \pi_j dj$ is total expenditures. The utilitarian government problem is therefore

$$S := \max_{\tau_x}\ e^{\tilde{\Omega}} Y_a^{\zeta}(1+\tau_m)^{\zeta-1}E^{1-\zeta}$$

or

$$\log(S) = \max_{\tau_x}\ \tilde{\Omega} + \zeta\log(Y_a) + (\zeta-1)\log(1+\tau_m) + (1-\zeta)\log(E).$$

where $\tilde{\Omega} := \Omega - \zeta\log(\zeta)$. By Lemma 2, E does not respond to τ_x. Therefore, the first order condition is

$$\zeta\frac{\partial\log(Y_a)}{\partial\tau_x} + (\zeta-1)\frac{\partial\log(1+\tau_m)}{\partial\tau_x} = 0. \tag{D.5}$$

By the implicit function theorem, we are interested in

$$\frac{\partial\tau_x^*}{\partial p_x^{\text{market}}} = -\frac{\frac{\partial^2\log(S)}{\partial\tau_x\partial p_x^{\text{market}}}}{\frac{\partial^2\log(S)}{\partial\tau_x^2}}$$

Since $\log(S)$ is concave,

$$\text{sign}\left(\frac{\partial\tau_x^*}{\partial p_x^{\text{market}}}\right) = \text{sign}\left(\frac{\partial^2\log(S)}{\partial\tau_x\partial p_x^{\text{market}}}\right). \tag{D.6}$$

Taking the derivative of (D.5) and using the results of Lemma 2 to note that $\partial\tau_m/\partial p_x^{\text{market}} = 0$, the sign restriction in (D.6) is equivalent to

$$\text{sign}\left(\frac{\partial\tau_x^*}{\partial p_x^{\text{market}}}\right) = \text{sign}\left(\frac{\partial^2 \log(Y_a)}{\partial\tau_x\partial p_x^{\text{market}}}\right). \tag{D.7}$$

Since

$$y_{aj} = \frac{\phi_j}{\gamma^\gamma}\left(\frac{\alpha_j}{p_x^{\text{market}}(1-\tau_x)}\right)^{\alpha_j}\left(\frac{\eta_j}{A}\right)^{\eta_j},$$

then

$$\log(Y) = \left(\frac{\nu}{\nu-1}\right)\log\left(\int_j \left[\Delta_j (p_x^{\text{market}})^{-\alpha_j}(1-\tau_x)^{-\alpha_j}\right]^{\frac{\nu-1}{\nu}} dj\right)$$

where Δ_j summarizes the remaining terms from y_{aj} that do not include p_x^{market} or τ_x. Taking the derivative with respect to the market fertilizer price,

$$\frac{\partial\log(Y_a)}{\partial p_x} = \frac{-1}{p_x^{\text{market}}}\int_j \alpha_j \frac{\left[\Delta_j (p_x^{\text{market}})^{-\alpha_j}(1-\tau_x)^{-\alpha_j}\right]^{\frac{\nu-1}{\nu}}}{\int_k \left[\Delta_k (p_x^{\text{market}})^{-\alpha_k}(1-\tau_x)^{-\alpha_k}\right]^{\frac{\nu-1}{\nu}} dk}.$$

Define the probability measure

$$\Gamma_j = \frac{\left[\Delta_j (p_x^{\text{market}})^{-\alpha_j}(1-\tau_x)^{-\alpha_j}\right]^{\frac{\nu-1}{\nu}}}{\int_k \left[\Delta_k (p_x^{\text{market}})^{-\alpha_k}(1-\tau_x)^{-\alpha_k}\right]^{\frac{\nu-1}{\nu}} dk}$$

so that we can re-write

$$\frac{\partial\log(Y_a)}{\partial p_x} = \frac{-\mathbb{E}_\Gamma[\alpha;\tau_x]}{p_x^{\text{market}}}$$

where $\mathbb{E}_\Gamma$ is the expectation of α taken using Γ, which depends (in part) on τ_x. This implies

$$\text{sign}\left(\frac{\partial^2 \log(Y_a)}{\partial\tau_x\partial p_x^{\text{market}}}\right) = -\,\text{sign}\left(\frac{\partial\mathbb{E}_\Gamma[\alpha;\tau_x]}{\partial\tau_x}\right). \tag{D.8}$$

Note that

$$\frac{\partial\log(\Gamma_j/\Gamma_k)}{\partial\tau_x} = \frac{\nu-1}{\nu}(\alpha_j-\alpha_k)\left(\frac{1}{1-\tau_x}\right).$$

If $\nu > 1$, this is positive for $\alpha_j > \alpha_k$, implying that weight is shifting toward higher α, and therefore $\partial\mathbb{E}_\Gamma[\alpha;\tau_x]/\partial\tau_x > 0$. (D.8) therefore implies $\frac{\partial^2\log(Y_a)}{\partial\tau_x\partial p_x^{\text{market}}} < 0$. The same exercise gives the required result that $\frac{\partial^2\log(Y_a)}{\partial\tau_x\partial p_x^{\text{market}}} = 0$ when $\nu = 1$ and $\frac{\partial^2\log(Y_a)}{\partial\tau_x\partial p_x^{\text{market}}} > 0$ when $\nu < 1$. By (D.7), $\partial\tau_x^*/\partial p_x$ shares the same signs.

■

D.5 Proof of Proposition 4

For simplicity, denote $\tilde{\phi}(s) = \phi s + \mathcal{W}_j$ as the maximum cost level faced by a household with s savings in village j throughout this proof, and the unit cost

$$c_{Mj} = \left(\alpha_j p_x^{1-\sigma} + (1-\alpha_j) w_{aj}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

We start with an additional lemma.

Lemma 3. *For any level of savings s, a household in village j chooses the modern technology if*

$$z \geq A_T \left(\frac{c_{Mj}}{w_{aj}}\right)^{\gamma}$$

Proof. Traditional profit is

$$\pi_T = \begin{cases} (pT)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} (1-\gamma), & \text{if } 1 \leq \frac{\phi^{1-\gamma} w^{\gamma}}{\gamma p T} \\ (pT)\left(\frac{\phi}{w}\right)^{\gamma} - \phi, & \text{otherise} \end{cases}$$

and modern profit is

$$\pi_M(z_M) = \begin{cases} (1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} (pz)^{\frac{1}{1-\gamma}} c_M^{\frac{-\gamma}{1-\gamma}}, & \text{if } z_M \leq \overline{z}^M \\ pz\phi^{\gamma} c_M^{-\gamma} - \phi, & \text{otherwise} \end{cases}$$

where c_M is the unit cost function $c_M = (\alpha p_x^{1-\sigma} + (1-\alpha) w^{1-\sigma})^{\frac{1}{1-\sigma}}$ and the cut-off value for the financial friction is $\overline{z}^M = \phi^{1-\gamma} c_M^{\gamma} p^{-1} \gamma^{-1}$.

The main step here is to show that the cutoff value for using modern technology z^* does not depend on which constraints are binding. First, we look for a cutoff z^* if the financial friction does not bind for both traditional and modern at z^*.

$$\begin{aligned} (1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}} (pz)^{\frac{1}{1-\gamma}} c^{\frac{-\gamma}{1-\gamma}} &\geq (pA_T)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} (1-\gamma) \\ \implies z &\geq A_T \left(\frac{c}{w}\right)^{\gamma} \end{aligned}$$

Similarly, if financial frictions are binding at the cutoff z^* for both technologies,

$$\begin{aligned} pz\phi^{\gamma} c^{-\gamma} - \phi &\geq (pA_T)\left(\frac{\phi}{w}\right)^{\gamma} - \phi \\ \implies z &\geq A_T \left(\frac{c}{w}\right)^{\gamma} \end{aligned}$$

The last step is to show that this is the only possible cutoff value. If at the cutoff value z^* we have a household financially constrained for the traditional technology but not the modern technology, the profit indifference condition is

$$pA_T\phi^{\epsilon}w^{-\gamma} - \phi = (1-\gamma)\gamma^{\frac{\gamma}{1-\gamma}}(pz^*)^{\frac{1}{1-\gamma}}c_M^{\frac{-\gamma}{1-\gamma}}$$

That the modern sector is unconstrained implies

$$\begin{aligned} z^* &\leq \overline{z}^M = \phi^{1-\gamma}c_M^{\gamma}p^{-1}\gamma^{-1} \\ \implies (z^*)^{\frac{1}{1-\gamma}} &\leq \phi c_M^{\frac{\gamma}{1-\gamma}} p^{\frac{-1}{1-\gamma}}\gamma^{\frac{-1}{1-\gamma}} \end{aligned}$$

Combining these two conditions yields

$$\gamma pTw^{-\epsilon} \leq \phi^{1-\epsilon}$$

which is a contradiction of the fact that the household is constrained in the traditional technology. A nearly identical procedure implies that there can be no cut-off value for any household in which the household is constrained in the modern technology but not the traditional one. Thus,

$$z^* = A_T\left(\frac{c_M}{w}\right)^{\gamma}$$

is the only feasible cutoff for any household in village j. ■

After this, the proof of Proposition 4 follows almost immediately, because we can rely on the usual properities of the Pareto distribution in the unconstrained profit maximizing case.

Proof of Proposition 4

Proof. By the properties of the Pareto distribution

$$S_{jt} = Pr(z > z_{jt}^*) = \left(\frac{z_M}{z_{jt}^*}\right)^{\theta_m} \implies \Delta\log(S_{jt}) = -\theta\Delta\log(z_{jt}^*)$$

and

$$\mathbb{E}[\log(z)|\text{use modern}] = \log(z_{jt}^*) + \frac{1}{\theta_M} \implies \Delta\mathbb{E}[\log(z)|\text{use modern}] = \Delta\log(z_{jt}^*)$$

Combining these two relationships and averaging across villages gives the required result. ■

Appendix References

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